Generative Art from the Calculus Classroom

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Popular tools for producing art with mathematics either tend to centre on design, and require little mathematical knowledge, or focus on mathematics and plotting tools, for example Matlab [1], Mathematica [2], Desmos [3]. The ideas explored here move from foundational mathematics through art, to tell a story about some of the basic concepts taught in a calculus course, including tangents, normal, radius of curvature, level sets and gradients. These explorations should be accessible to people who have not taken an advanced calculus course. We start from a basic diagram of the concept in question, as it might appear in a textbook, and move stepwise from the mathematics to an artistic expression based on an understanding of the concept. The art produced aims to be a playful expression of mathematics, inviting further thought on both the mathematical ideas and the potential for artistic exploration. In addition, I want to single out simple concepts and show how to enjoy them in their own right, as opposed to the teacher focusing on the goal of use of the concepts, in engineering, medicine, and countless other fields, which may be too distant for some beginning students. Topics can be mastered simply by computing plenty of examples by hand, but not all students will find this sufficiently enjoyable to hold their interest. Mathematics is not always taught with as much emphasis on the creative process as is necessary for the discovery of new mathematics. Introducing visual art may help some students find their path to creativity in mathematics. The humble tangent line is just a very first step on the way to being able to solve important problems necessary in the use of technology in modern life. Sometimes it seems that there is a huge gulf between those who "get it" and quickly move on from the basics to the more advanced applications, and those who never really master the first steps in calculus. The invitation to consume and make artwork based on the basic concepts is an invitation to join the conversation, and perhaps contribute to these fields. Making an artwork based on the first concepts in calculus causes one to pause and savour the constructions. Using art may help students who do not immediately grasp a lesson. Generative art can help the teacher convey her understanding of and enthusiasm for the topic, and also slow down the presentation, which may help some students. Art can make the lesson more enjoyable, by making the inherent creativity in teaching and doing mathematics more visible to those students who do not at first see this.

1. Tangent lines

The tangent line is one of the most basic concepts in calculus. A beginning text might contain a diagram as in Figure 1, (latex tikzpicture) showing a tangent just touching a curve.

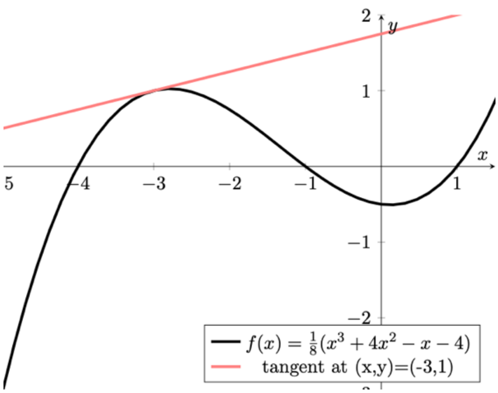

Figure 1: Textbook example: tangent at a point.

The use of tangents in string art is well known [4], for example Figure 2 shows lines forming tangents to a parabola, and Cremona's construction of a cardioid curve (classic 19th century algebraic geometry). Many examples are available in the online program GeoGebra [5]. Tangents to a sine curve are shown in Figure 3 (fixed length) Figure 4 (sinusoidally varying length). Animating the curves and tangents produces even more possibilities [6].

Producing such figures, and predicting the outcome from the equations, could be a good exercise for beginning calculus students.

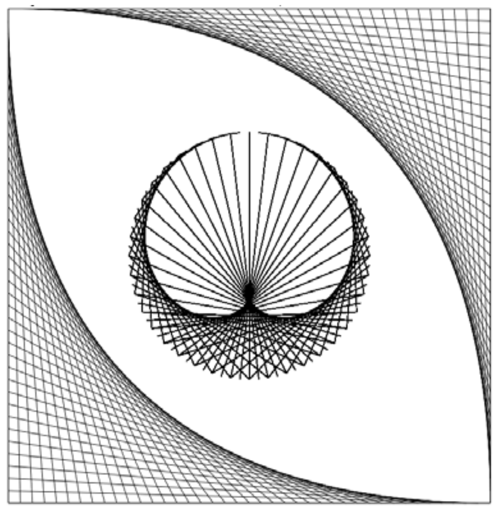

Figure 2: Tangent line string art.

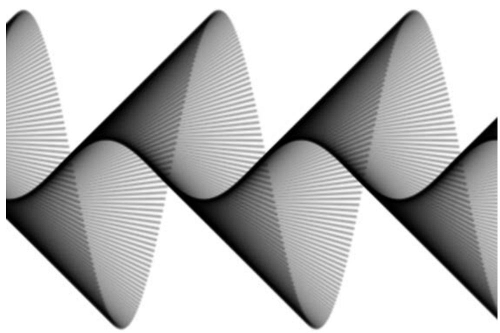

Figure 3: Tangents to y=sin(x).

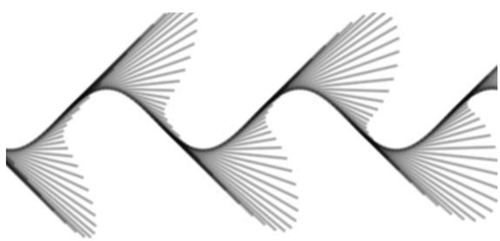

Figure 4: Variable length tangents to y=sin(x).

Producing such figures, and predicting the outcome from the equations, could be a good exercise for beginning calculus students.

2. Normals

From tangent lines, we pass to normals. The normal vector at a point on a curve is a line perpendicular to the tangent, as shown in the diagram in Figure 5.

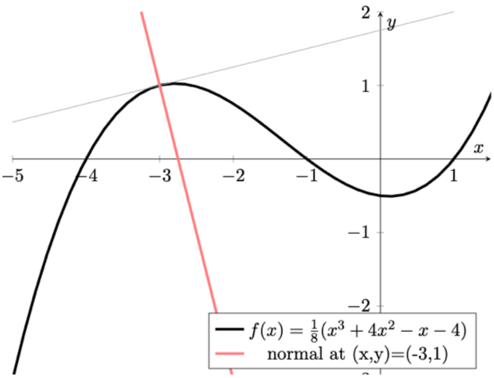

Figure 1: Textbook example: normal at a point.

Normal vectors to the sine curve are shown in Figure 6. The lengths in this example are given by the radius of curvature.

2. Radius of curvature

Whereas a tangent line is a *line* that just touches a curve, the *osculating circle* is a *circle* which just touches the curve and is as big as possible without crossing the curve, as illustrated in Figure 7.

The radius of the osculating circle is the *radius of curvature*. F igure 8 shows many osculating circles to a sine curve. The number of circles, their placement, and distribution will produce very different effects from the same curve, for example, compare Figures 8 and 9. Figure 9 is more “educational”: the viewer is invited to consider which circle is an os culating circle at which point on the curve; this is

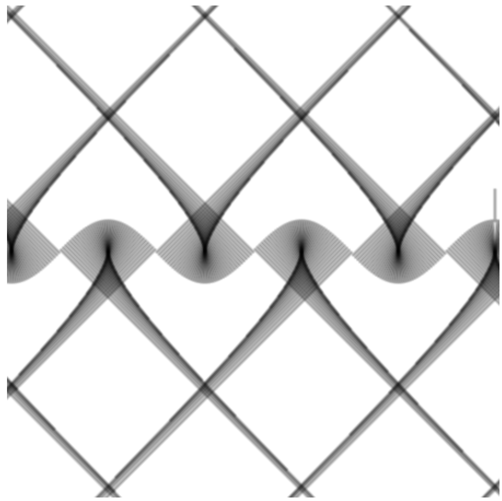

Figure 6: Normals to y=sin(x).

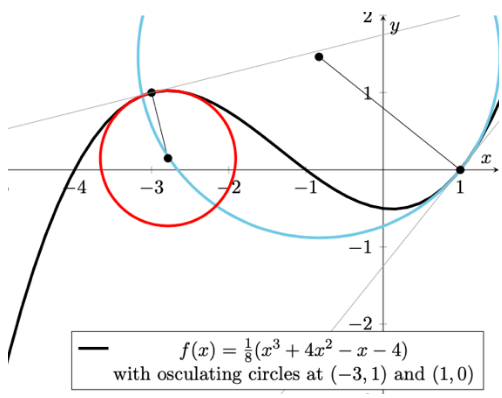

Figure 7: Textbook example: osculating circles at (-3,1) and (1,0).

harder to see in Figure 8. Figure 10 combines normals and osculating circles.

The osculating circle has centre on the normal to the curve, at a distance from the curve given by the radius of curvature. Playing around with these ideas, we could draw the circles with the correct centre, but change the radius of the circles, for example, as in Figure 11. Many variations are possible.

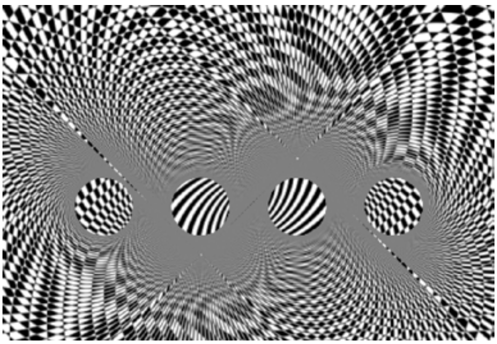

Figure 8: Osculating circles to y=sin(x)

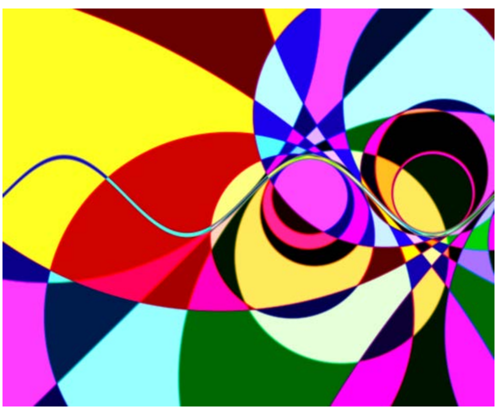

Figure 9: Osculating circles to y=sin(x).

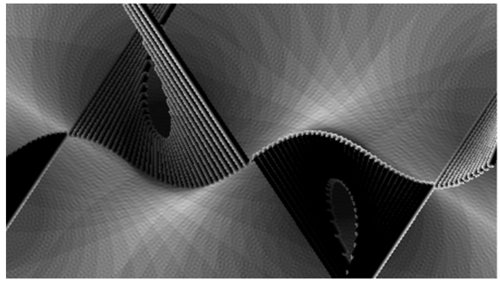

Figure 10: Normals & Osculating circles.

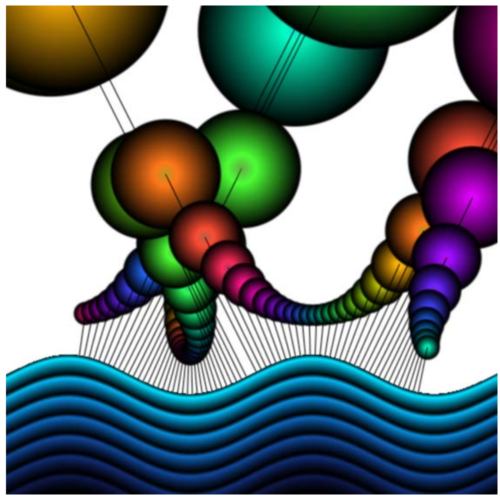

Figure 11: Based on osculating circles

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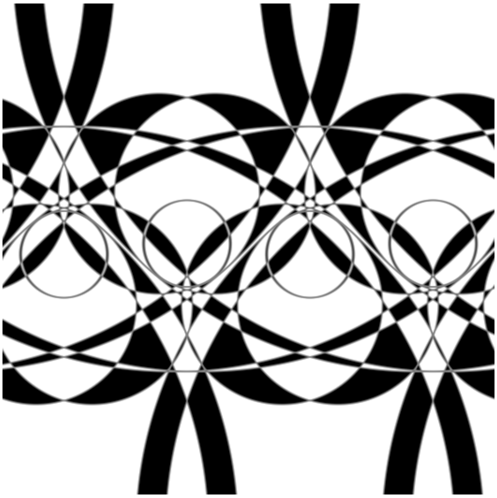


Figure 12: Osculating circles of y=sin(x)

3. Level Sets and Gradient

Two of the first concepts learned in a multivariate calculus class are level sets and gradient. For a function f(x,y) and a constant c, the level set $L_c(f)$ is the set of points (x,y) where f(x,y)=c. In the context of maps these are called contours. The level sets form a collection of curves, as shown by the red curves in Figure 13. The gradient is a vector field, that is, a collection of vectors, which in the map interpretation, will point uphill. These are shown as blue arrows in Figure 13.

Further examples are shown In Figures 14 and 15. A vector field is a specification of a vector at every point in the domain for example, a wind speed at every point. But we can only draw a representative selection of vectors.

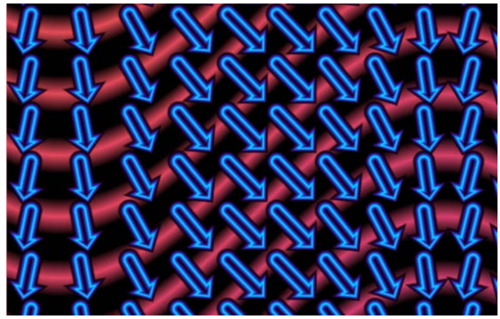


Figure 13: Level sets and gradients

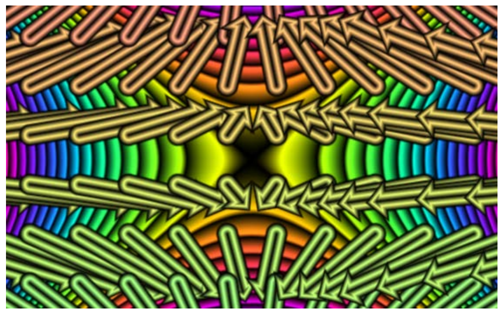


Figure 14: Level sets and gradients

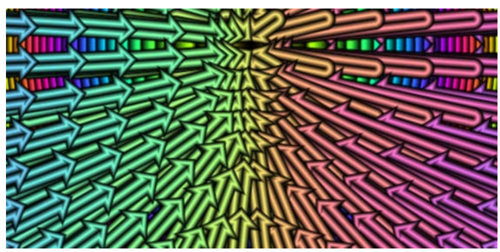


Figure 15: Gradient vectors.

In Figures 16 and 17, the gradient vectors are replaced by lines perpendicular to the contour lines, which are in the direction of the gradient vectors at any point, so produces a levels/gradient field effect.

Figure 16: Generative art from a gradient.

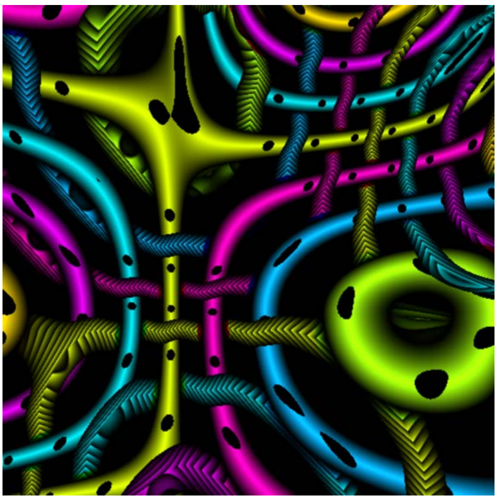


Figure 17: Generative art from a gradient.

5. Conclusion

These generative art works aim to convey calculus ideas in a memorable, non-intimidating way to beginning students. The idea is to complement a standard textbook such as [7] with some light relief, and food for thought. These figures can be animated for enhanced effect. In future I intend to create work based on concepts such as divergence, curl, and theorems taught in a first multivariate calculus course.

References

[1] MatLab: matlab.mathworks.com

[2] Mathematica: wolframalpha.com

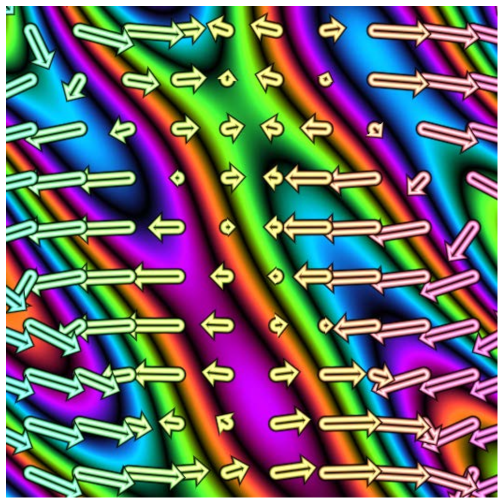
[3] Desmos: desmos.com

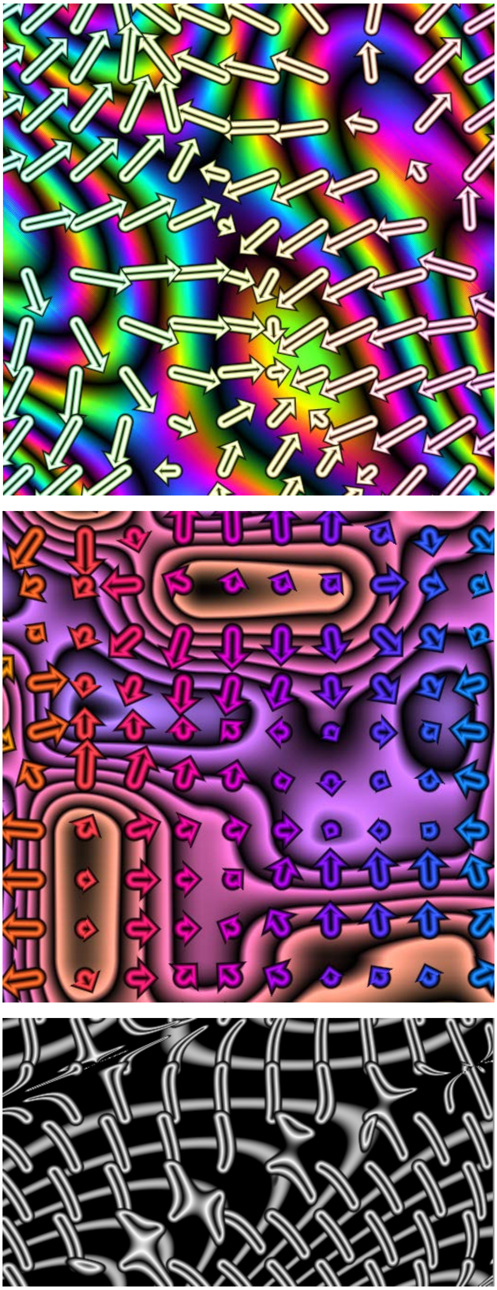
[4] C. von Renesse & V. Ecke (2016) Discovering The Art of Mathematics: Using String Art to Investigate Calculus, PRIMUS, 26:4, 283-296.

[5] GeoGebra, http://www.geogebra.org

[6] H. A. Verrill, www.mathamaze.co.uk/circles/geometryundergrad/

[7] J. Stewart, Multivariate calculus, 7th edition, ISBN: 9780538497879





Figures 18-21: levels sets and gradient examples.