Periodic Structures on Phyllotactic Patterns

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Abstract

The aim of this note is to show the possibility of reflecting some periodic and quasiperiodic sequences on the phyllotatic pattern.

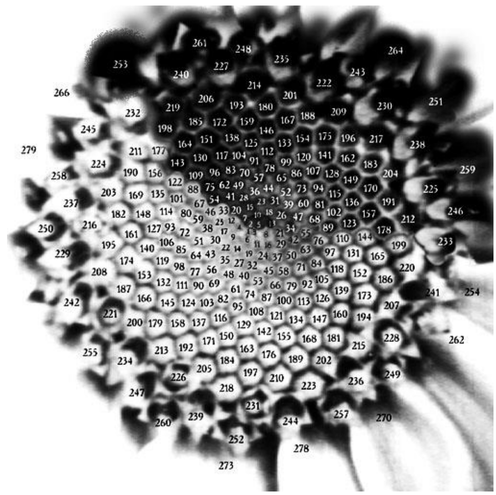

Fig. 1. The projection of two complementary Beatty sequences on the set of florets of a chamomile conflorescence.

Our approach proposes the distribution of the spectrum of a real number on a polar integer lattice. Plotting on the basis of this principle is considered as a tool for visualizing and studying various phenomena associated with periodicity, be it the movement of celestial objects or the rules of musical harmony. The representation of the spectrum of a number in the polar coordinate system is carried out, in addition, in an analytical (algebraic) form, a histogram and the so-called Bresenham line.

1. Introduction

Everywhere we observe the periodic phenomena, for example, change of day to night, seasons of year, etc. Usually for the image of these time phenomena we use rectangular or circular tables (calendars, dial of hours).

In the general words, rectangular tables are habitual for perception, they transfer originality of varying years, but graphically cycle form, i.e. a circle symbolizing repetition, is lost.

The circular tables are closed, and all periods appear similar to one another.

The spiral form, in a sense, is intermediate. It unites advantages and levels lacks of both forms mentioned above. However, the spiral table is not habitual and is not so easy for perception.

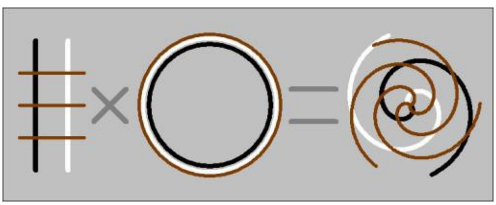

Fig. 2. The spiral shape can be thought of as a hybrid of a rectangular grid and a circle.

Modular arithmetic is the reliable tool for work with calendars.

This paper has its origins in the conception of phyllotaxis. Phyllotaxis studies the symmetrical (asymmetrical) constructions determined by organs and parts of plants. "New light has been cast on the subject with the realization that phenomena similar to phyllotaxis occur in realms outside of botany" [1].

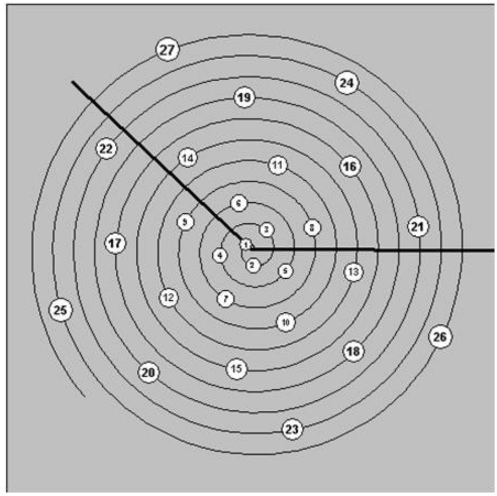

Fig. 3. The plots in the polar coordinate system are united by location of the points of the spiral with a given angle of divergence.

We will conventionally designate this coordinate system as *phyllotactic*. Such concepts as the spectrum of a real number, Beatty sequences inclusively, continued fractions, Euclid's algorithm, two-dimensional crystalline projection, Bresenham's line, the spirographs theory come into view.

The periodic sequences in the polar coordinate system are often depicted in one dimension. Only the angular component with a unit radius is taken into account (the clock face, the spirograph theory, musical rhythms based on the Euclidean algorithm, the circle of fifths). In the phyllotactic system, both parameters – an angle from a reference direction and a distance from a reference point – are variable. We believe that the Archimedean and Fermat's spirals are convenient for this purpose.

2. The spectrum of a real number

The spectrum of a real number α is defined to be an infinite multiset of integers.
Spec(α) = {⌊α⌋,⌊2α⌋,⌊3α⌋, …}; ⌊x⌋ = the greatest integer less than or equal to x (floor); [2]. If α is an irrational number, then Spec(α) is called a Beatty sequence. If α and β are positive irrational numbers such that 1/α + 1/β = 1, then the Beatty sequences ⌊α⌋, ⌊2α⌋, ... and ⌊β⌋, ⌊2β⌋, ... together contain all the positive integers without repetition.

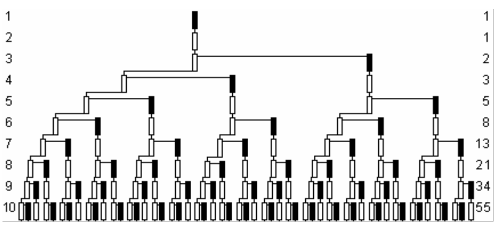

Fig. 4. Genealogical tree of Fibonacci rabbits. Fibonacci chain.

There is a remarkable sequence of 0s and 1s, which is intimately related to the Fibonacci numbers and to Phi. There are different ways to generate this Fibonacci chain or Rabbit sequence.

The Substitution Map

0 → 0 1

1 → 0

gives

0 → 0 1 → 0 1 0 → 0 1 0 0 1 → … ,

giving rise to the sequence

0 1 0 0 1 0 1 0 0 1 0 0 1 0 1 0 0 1 0 1 0 0 1 0 0 1 0 1 … .

Here, the zeros occur at positions

1, 3, 4, 6, 8, 9, 11, 12, … ,

and the ones occur at positions

2, 5, 7, 10, 13, 15, 18, ….

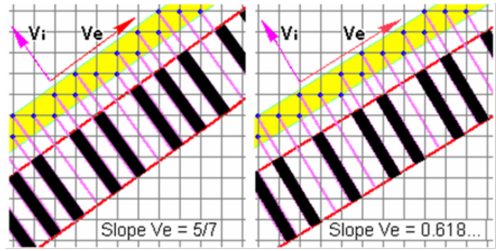

Fig. 5. Projection of a two-dimensional lattice onto a one-dimensional space with a rational slope (5/7) (left) and an irrational slope ((√5)-1)/2) (right) to obtain a one-dimensional approximant and quasicrystals, respectively. If the slope of the vector Ve, equal to the tangent of its angle with the horizontal, is rational number, then the sequence turns out to be periodic. If the slope is irrational, then, respectively, the sequence is non-periodic, or quasi-periodic.

The concept of the spectrum of a real number which is closely related to Bresenham's "midpoint line algorithm" and Euclid's algorithm finds its way into calendrical calculations. *Bresenham's line algorithm* is a line drawing algorithm that determines t he points of an *n*-dimensional raster that should be selected in order to form a close approximation to a straight line between two points. Dershowitz et al. derive some general formulas that are useful in calendar conversions for the Julian, Islamic, Coptic, Hebrew, arithmetic Persian, and old Hindu lunisolar calendars [3]. We illustrated some calendar systems in phyllotactic representation with computer animations. Among them there is the Runic calendar, the 532-year "Victorian" or "Dionysian" cycle for the date of Orthodox Easter, Julian Easter perpetual calendar which allows you to define the day of the week and the day of Orthodox Easter.

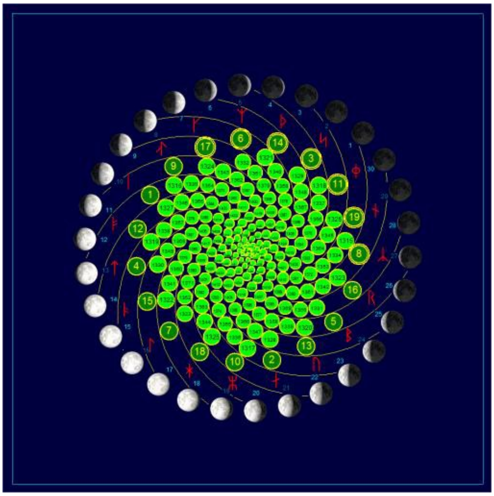

Fig. 6. Phyllotactic pattern of Metonic Cycle. Scandinavian lunar calendar.

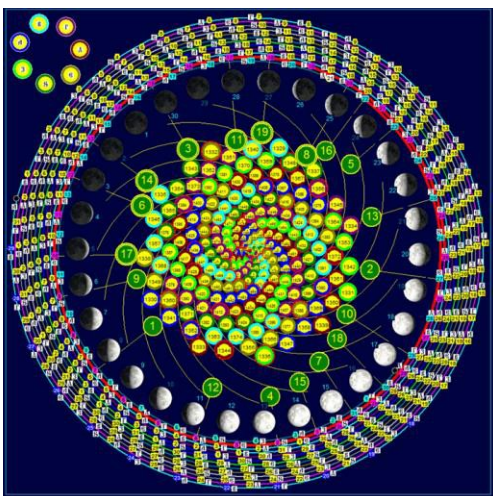

Fig. 7 Julian Easter perpetual calendar.

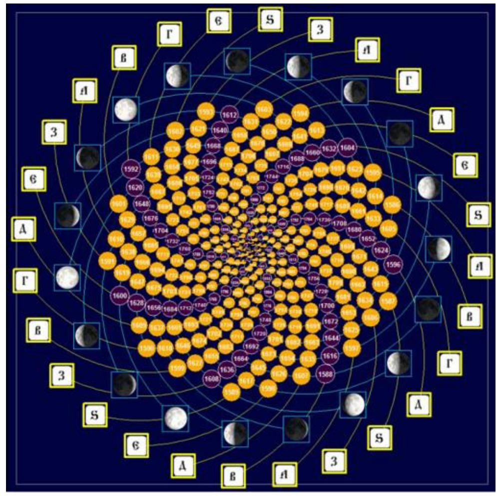

Fig. 8. The 532-year "Victorian" or "Dionysian" cycle.

The spectra of the numbers 12/7 and 12/5 provided a pattern of piano keyboard, and their projection onto the phyllotactic sample with divergence angle 360°*5/12 = 150° gave a two-dimensional layout of a one-dimensional circle of fifths.

The figures below show four ways to graphically represent the spectrum of a real number. In each illustration, the upper left quadrant shows phyllotactic pattern; upper right quadrant shows keyboard one (similar to a piano keyboard); lower left quadrant shows the analytic representation; lower right quadrant matches Bresenham's line algorithm. The sequence of the ratio of neighboring Fibonacci numbers, musical fifth (7/12), the Leap years cycle and the Metonic cycle are chosen for example.

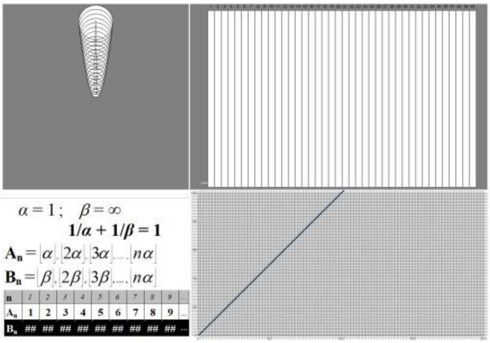

Fig. 9. 1/ α = 1 / 1

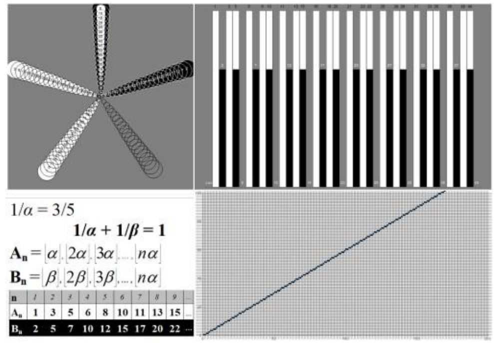

Fig. 12. 1/ α = 3 / 5

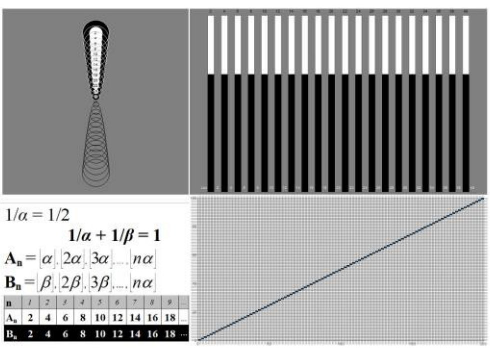

Fig. 10. 1/ α =1 / 2

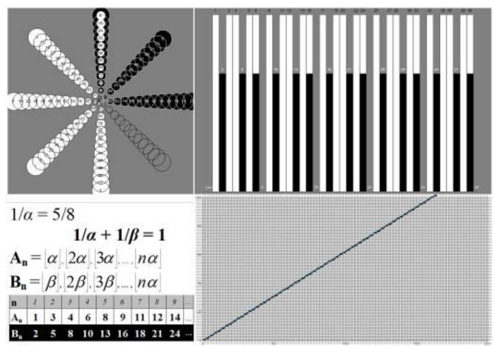

Fig. 13. 1/ α = 5 / 8

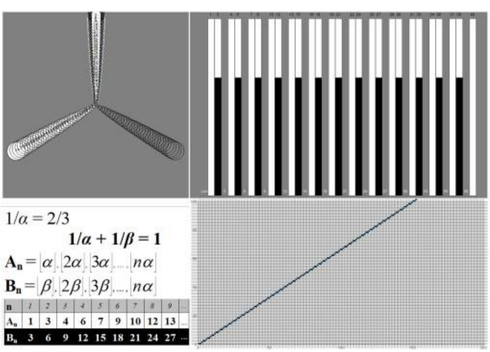

Fig. 11. 1/ α = 2 / 3

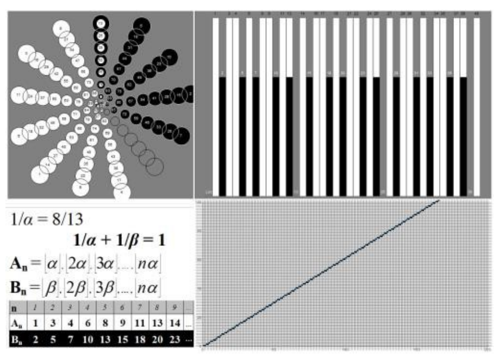

Fig. 14. 1/ α = 8 / 13

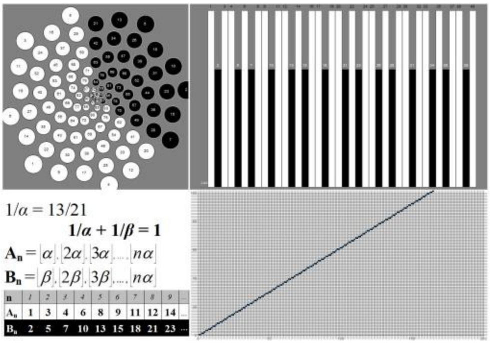

Fig. 15. 1/ α = 13 / 21

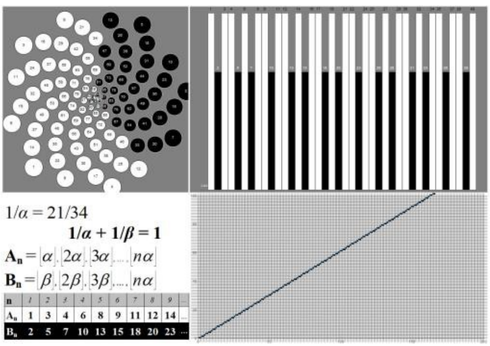

Fig. 16. 1/ α = 21 / 34

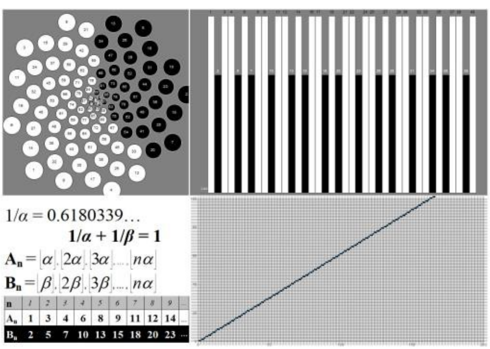

Fig. 17. 1/ α = 0.6180339…

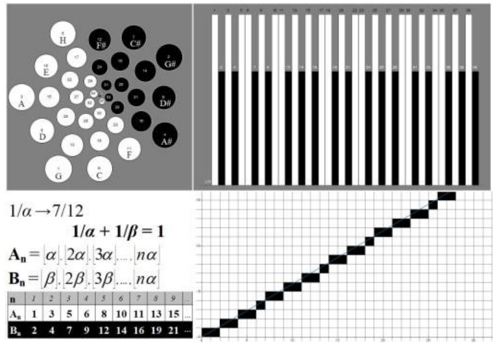

Fig. 18. Circle of Fifths

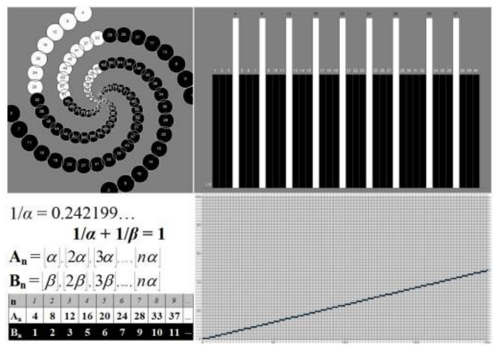

Fig. 19. Leap years

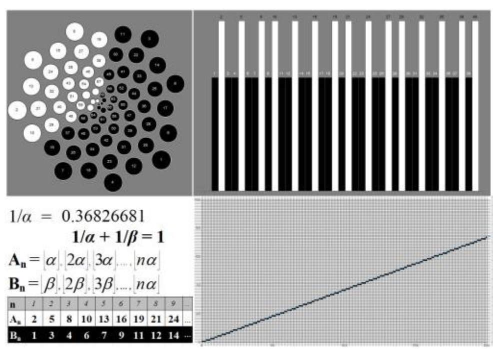

Fig. 20. Metonic Cycle

3. Simultaneous Cycles

“Some calendars employ two cycles running simultaneously.” [3].

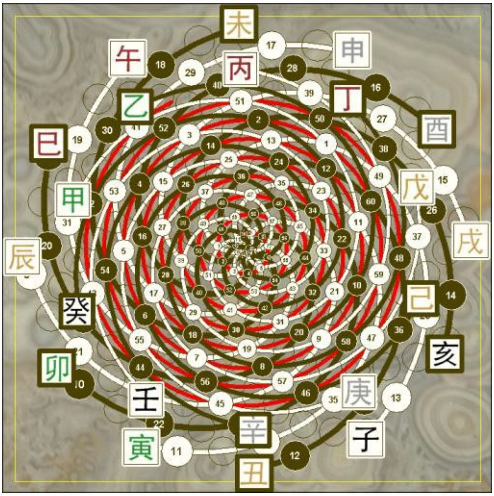

Fig. 21. Chinese calendar 60-year cycle.

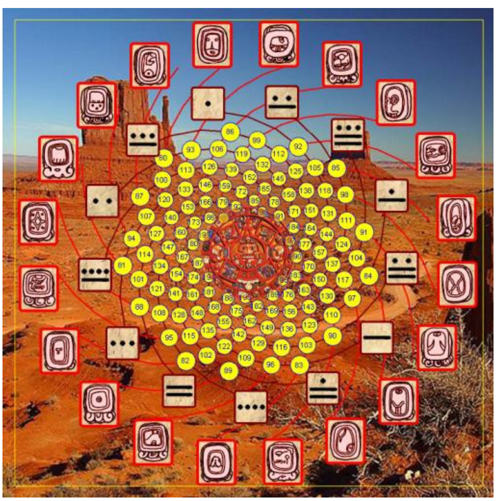

Fig. 22. The Tzolkin calendar.

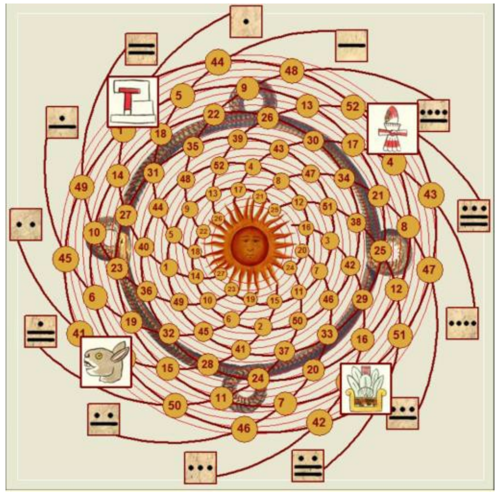

Fig. 23. The Xihuitl, The Cyclic 52-Year Calendar.

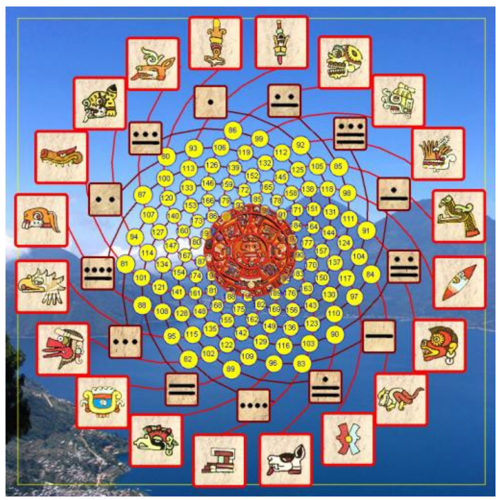

Fig. 24. Tonalpohualli, meaning “count of days” in Nahuatl, is a Mexica version of the 260-day calendar in use in pre-Columbian Mesoamerica.

Fig. 25. "Daariisky Krugolet Chisloboga". The phyllotactic pattern 9 x 16 = 144.

4. Conclusion

On the one hand our article shows that phyllotactic patterns can be used to illustrate known structures in a new way, by transposing them into another coordinate system. On the other hand, the projection of integer sequences onto the phyllotaxis lattice is in line with the Cognitive Visualization concept proposed by Alexander Zenkin. "Cognitive Visualization aims to represent an essence of a scientific abstract problem domain, i.e. the most principal connections and relations between elements of that domain, in a graphic form in order to see and discover an essentially new knowledge of a conceptual kind" [4].

5. References

[1] Adler, I., Barabe, D., Jean R. V. (1997). *A History of the Study of Phyllotaxis*. Annals of Botany 80(33), 231-244. (p. 231)

[2] Graham, R. L.; Knuth, D. E.; Patashnik, O. (1994). *Concrete Mathematics: A Foundation for Computer Science,*(2nd ed.), Addison-Wesley, xiii, 657, ISBN 0-201-55802-5. (p. 67, 77)

[3] Dershowitz, N., Reingold, E. M. (2008). *Calendrical Calculations,* (3nd ed.). © Cambridge University Press, ISBN 978-0-521-88540-9.

[4] Zenkin, A., (2010). *Cognitive (Semantic) Visualization of the Continuum Problem and Mirror Symmetric Proofs in the Transfinite Numbers Theory* https://vismath1.tripod.com/zen/zen1.htm