

Generative Design under the Intelligent Manifold Coordinate Systems

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“perfect” geometric proportions phenomenon for development Generative Design under the intelligent manifold coordinate systems. We would like see grandiose spatial harmony that as known, presented in the world surrounding of us as the fundamental laws of nature that exist for ever. Applicability these laws able to represent of the perfect geometric proportions in a reasonable way, e.g., using vector data coding, processing, computing, and systematic researches founded on the idea of minimizing vector basis of the intelligent coordinate system. We use linear combinations over the basis taking in account t -modular sums of this basis to cover the manifold coordinate system reference grid exactly R -times. These designs involve techniques for improving the quality indices of vector information technologies and artificial systems with two- and multidimensional structures, using vector data processing with respect to big vector data content, computing speed, memory reducing, data transfer rate, information redundancy, and code security communication. Systemic

Abstract

This paper presents the intelligent manifold coordinate systems for generative design under the system based on the idea of “perfect” geometric proportions of spatially or temporally distributed elements (events) as parts of “whole” and logical interpretation of the phenomenon including philosophical aspect for better understanding role of geometric structures in behaviors of natural and artificial objects. It was graphically that the “intelligent” rotational symmetry and asymmetry mutual penetration is an example existing of eternal world intelligence of the Universe. This make it possible to use the the

researches based on remarkable geometric properties of space is that a t -dimensional metric space can be represented as t -modular manifold coordinate system of t ring axes with common reference ground. Moreover, we require the set of all manifold coordinate grid node points is one-to-one a set of the indexed big data attributes and categories, as well as the set of all binary code combinations that has been created by this basis. We can use binary optimized t -dimensional vector codes both non-redundant and code protective monolithic ones. The first of them provides optimum encoding design, compression and processing big vector data with respect to code sizes, while the second is self-correcting code. These techniques involve modular arithmetic for computing and transforming procedures under manifold coordinate system, using appropriate algebraic constructions, such as Ideal Ring Bundles (IRB)s cyclic groups and Galois field theory. IRBs originate from the "intelligent" rotational symmetry and asymmetry relationships of space, known as the concept of Perfect Distribution Phenomenon (PDP), which create intelligent relationships "parts- whole" of complementary asymmetries joined harmonically in the rotational symmetry, forming intelligent manifold t -dimensional coordinate system of the manifold shape. Proposed approach offers the development of multi-dimensional optimum vector data processing and coding design with direct applications to intelligent information technologies and big data processing.

1. Introduction

The main aims of the generative approach are the progression and the multiplicity [1]. The main problem of modern information technology is development of effective vector data processing for finding optimal solution of wide classes of problems, including large data amounts analysis. However, the design based on the traditional theory is not always applicable for multidimensional data processing. In general case it was possible to take in consideration a new conceptual model of the data processing, based on the laws of worldwide harmony, such as Golden ratio and Perfect Distribution Phenomenon [2]. The problem to be of very important for configure multidimensional data processing with fewer structural elements and bonds than at present, while maintaining or improving on simplicity and the other operating characteristics of the data processing. These techniques are profitable for development of fundamental and applied researches in multi-modular optimum coding systems [3], non-redundant space-tapered arrays of radar or sonar systems [4], chemical physics [5], and manufacturing [6]. The concept associates with Perfect Distribution Phenomenon (PDP), which is that a unity can be partitioned "perfectly" in ring sequence form, and the sums of connected sub-sequences of a unity enumerate the set of integers exactly R -times. This property makes PDP extremely effective, when applied to the problem of finding the optimum ordered events in spatially or temporally distributed systems. Applications profiting from PDP are for example one- and multidimensional vector data coding

design [3]. The idea of partition sets with the smallest possible number of intersections is in agreement with describing the physics of toroidal confined plasmas [7]. In one dimension, a usual single-holed torus is the 1-torus [8] as a ring shape object. In two dimensions, we see a usual torus, also called the 2-torus. In analogy with this concept, in three and more dimensions, the t -dimensional torus, or t -manifold [9] is an object that exists in dimension $t+1$. This notion used to help visualize aspects of higher dimensional toroidal spaces. It was the torus mathematical model useful for describe geometric objects in spatial coordinates. The torus topology is superior to geometry for describing such objects because relate with philosophical spatial relationships. Many scientists also have suggested that the entire universe is a torus. Modern geometry is the study of spatial structures using Galois algebra [10], projective geometry [11], and combinatorial theory [12].

2. The intelligent symmetry and asymmetry ensembles

"Symmetry, as wide or as narrow as you may define its meaning, is one idea by which man through the ages has tried to comprehend and create order, beauty and perfection." H. Weyl

2.1 Symmetry and asymmetry intelligent relation

Symmetry and asymmetry relation in geometric structure is the most familiar type of them. The more general meaning of symmetry-asymmetry is in combinatorial configurations as a whole. In this context, symmetries and asymmetries underlie some of the most

profound results found in modern physics, including aspects of space and time [13]. Finally, discusses interpenetrating symmetry and asymmetry in the humanities, covering its rich and varied use in architecture, philosophy, and art. Space-time symmetries are features of space-time that can be described as exhibiting some form of symmetry [14]. The role of symmetry in physics is important in simplifying solutions to many problems, e.g. exact solutions of Einstein's field equations of general relativity [15], and study of isometrics in two or three-dimensional Euclidian space [16]. Only one angular interval in one-fold rotational symmetry enumerates the set {1} exactly once ($R=1$) is singleton, known as a unit set [17].

Let us regard a sketch of S -fold rotational symmetry joined on two complementary asymmetries of the symmetry, where we require all angular distances between of straight lines emanated from a common point in each of the complementary asymmetries enumerate the set of angles fixed number of times. An example of such rotational symmetry of order seven ($S=7$) given in Fig.1.

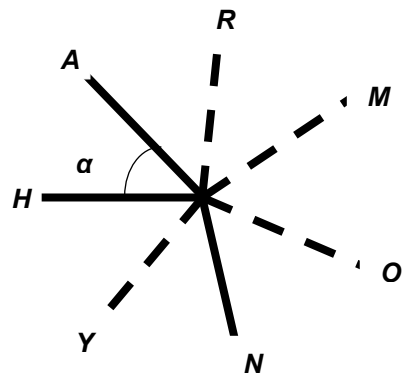


Fig. 1 A sketch of rotational symmetry of order seven (S=7) joined on two complementary asymmetries of the symmetry represented by three solid and four broken lines

5α	Y	O
6α	O	M

Hence, the ring scale reading system based on 7-fold (S=7) rotational symmetry allows on partition two-dimensional space perfectly for minimum number of intersections relative to reading point by spatial interval $\alpha=360^\circ/7$ exactly once ($R_1=1$) and/or twice ($R_2=2$) by the same interval. We call this “intelligent symmetry and asymmetry ensemble” of order S=7.

If we allow go round seven (S=7) lines of the 7-fold rotational symmetry, moving clockwise reference points **HARMONY** (Fig.1), we can obtain a set of angular distances $[\alpha, 6\alpha]$ between of distinct pairs of three ($n_1=3$) broken lines, $\alpha=360^\circ/S = 360^\circ/7$ (Table 1). $n_2=4$ solid and four ($n_2=4$)

Easy to see, that the 7-fold rotational symmetry creates intelligent ensemble of two complementary numerical ring structures {1, 4, 2}, and {1, 1, 2, 3}, followed from **H→A→N→H**, and **R→M→O→Y→R** cyclic sequences, and number of ensembles generated by S – fold rotational symmetry is theoretically uncountable. Easy-to-interpret sketch of the intelligent system based on 7-fold rotational symmetry and asymmetry ensemble depicted in Fig.2.

Table 1. Ring scale reading rotational symmetry as cyclic coordinate system with seven (S=7) reference points **HARMONY**, moving clockwise

Angle	Starting point	Final point
α	H	A
2α	N	H
3α	N	A
4α	A	N
5α	H	N
6α	A	H
Angle	Starting point	Final point
α	R	M
2α	R	O
3α	M	Y
4α	R	Y
5α	O	R
6α	M	R
Angle	Starting point	Final point
α	M	O
2α	O	Y
3α	Y	R
4α	Y	M

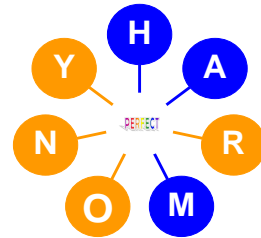


Fig. 2 Easy-to-interpret sketch of the intelligent system based on 7-fold rotational symmetry and asymmetry ensemble.

Parameters of the intelligent systems based on the S-fold rotational symmetry and asymmetry ensembles for $3 \leq S \leq 31$

presented in Table 2.

Table 2. Parameters S , n_1 , R_1 , n_2 , R_2 of the intelligent systems based on the S -fold rotational symmetry and asymmetry ensembles for $3 \leq S \leq 31$.

S	n_1	R_1	n_2	R_2
3	1	1	2	1
7	3	1	4	2
11	5	2	6	3
13	4	1	9	6
15	7	3	8	4
19	9	4	10	5
21	5	1	16	12
23	11	5	12	6
31	6	1	15	7

To see Tabl.2, we observe, that S -fold intelligent systems based on the S -fold rotational completed both from asymmetries of even (n_1), and odd (n_2) orders, each of them allows an enumeration the set of all spatial intervals $[\alpha, 360^\circ]$ by step $360^\circ/S$ exactly R_1 or R_2 times. The favourable qualifies of the rotational symmetry makes it useful in applications, which need to partition two-dimensional ($t=2$) space or a set with the smallest possible number of intersections.

2.2 Ideal Ring Bundles

The ordered chain approach to the study of systems is known to be of widespread applicability, and has been extremely effective when applied to the problem of finding the optimum ordered arrangement of structural elements in a distributed technological system.

Let us calculate all C_n sums of the terms in the numerical n -stage chain sequence of distinct positive integers $\{k_1, k_2, \dots, k_n\}$, where we require all terms in each sum to be consecutive elements of the sequence. Clearly, the maximum such sum is the sum $k_1 + k_2 + \dots + k_n$ of all n elements:

$$C_n = 1+2+\dots+n = n(n-1)/2 \quad (1)$$

If we regard the chain sequence as being cyclic, so that k_n is followed by k_1 , we call this a ring sequence [3]. A sum of consecutive terms in the ring sequence can have any of the n terms as its starting point, and can be of any length (number of terms) from 1 to $n-1$. In addition, there is the sum of all n terms. Hence, the maximum number of distinct sums of consecutive terms of the ring sequence given by

$$S_n = n(n-1)+1 \quad (2)$$

Comparing the equations (1) and (2), we see that the number of sums S_n for consecutive terms in the ring topology is nearly double the number of sums C_n in the daisy-chain topology, for the same n terms. A more general type of IRB, where the $S_n - 1$ ring sums of consecutive terms give us each integer value from 1 to $n(n-1)$ exactly R times, as well as the value S_n (the sum of all n terms) exactly once. Here we see:

$$S_n = n(n-1)/R + 1 \quad (3)$$

Enumeration of IRB {1, 1, 2, 3} with $n=4$, $S_n=7$, $R=2$ typify of ring sum makes

Number	Ring sums of IRB {1,1,2,3}			
	1	1	2	3
1	+			
1		+		
2	+	+		
2			+	
3				+
3		+	+	
4	+	+	+	
4	+			+
5			+	+
5	+	+		+
6		+	+	+
6	+		+	+
7	+	+	+	+

evident Table 3.

Table 3. Enumeration of IRB {1,1,2,3} ring sums

The IRB {1, 1, 2, 3} allows an enumeration of ring sums from 1 to $S_n-1=6$ exactly twice ($R=2$), while the value $S_n=7$ exactly once.

One-dimensional IRBs are cyclic sequences of positive integers, which form perfect partitions of a finite interval $[1, S]$ of integers. The sums of connected sub-sequences of an IRB enumerate the set of integers of interval $[1, S-1]$ exactly R -times in a ring scale reading intelligent system of S -fold rotational symmetry.

To extract meaningful information from the underlying comparison let us apply to S -fold symmetry as a quantized planar field of two complementary completions

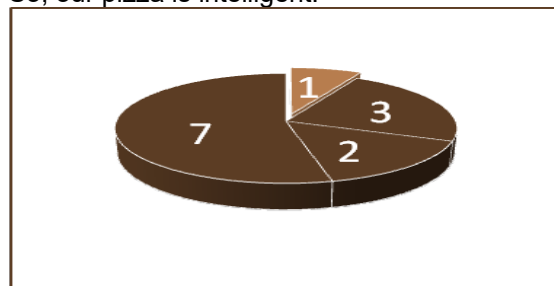
of the symmetric field. The remarkable properties of one-dimensional IRBs provide harmoniously quantization of 2D space with the smallest possible number of intersection, which make it possible to reproduce the maximum number of combinatorial varieties in the intelligent reference systems with a limited number of elements and bonds.

2.3 Poetry to the Intelligent Pizza

Our pizza is intelligent –
The numbers know all of it!
Ideally these numbers are all related in the ring.

Their chain sums add us new numbers.
The numbers are not the end,
As there are no ends in the ring.
And this idea lives in the orbits of IRB

That's the kind of pizza we have-
So, our pizza is intelligent!



1, 2, 3, 4=1+3, 5=3+2, 6=1+3+2, 7, 8=7+1, 9=2+7, 10=2+7+1, 11=7+1+3, 12=3+2+7, 13=1+3+2+7, 14=1+3+2+7+1, 15=2+7+1+3+2, 16=3+2+7+1+3, 17=1+3+2+7+1+3, etc.

Neither the beginning nor the end
You will find in the ring.
Our pizza is adorable-
The numbers know all of it!
Ideally these numbers are
All related in the ring.

The numbers are not the end,
 As there are no ends in the ring.
 Having understood the essence of a
 wonderful,
 Thyhamsh the code of the world-
 That's the kind of pizza
 We have- our pizza is !!!

3. Intelligent 2D coordinate system

Let us regard an n -stage ring sequence of two-dimensional vectors $\{(k_{11}, k_{12}), (k_{21}, k_{22}), \dots, (k_{i1}, k_{i2}), \dots, (k_{n1}, k_{n2})\}$, using 2D IRB as a basis for configure optimized toroidal coordinate system, where we require a set of all vector-sums combined on the basis, taking modulo m_1 and modulo m_2 accordingly, to be covered a torus by coordinate grid of sizes $m_1 \times m_2$. It is now customary to represent a toroidal coordinate systems as two-dimension reference grid of sizes $m_1 \times m_2$ covered surface of usual torus about two ($t=2$) concurrent reference axes with $(0,0)$ common point (Fig.3).

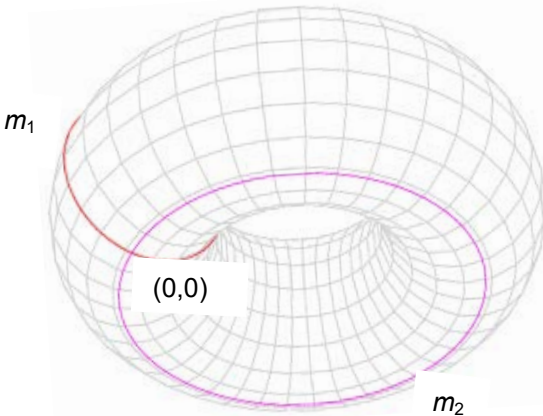


Fig.3 A toroidal coordinate systems as two-dimension reference grid of sizes $m_1 \times m_2$ covered surface of usual torus

about two ($t=2$) concurrent reference axes with $(0,0)$ common point

Example: The set of three ($n=3$) 2-stage ($t=2$) sequences $\{K_1, K_2, K_3\}$, $K_1=(0,2)$, $K_2=(0,1)$, $K_3=(1,2)$, forms two-dimensional ($t=2$) coordinate grid $m_1 \times m_2 = 2 \times 3$ taking the double modulo (mod 2, 3) sums of the IRB $\{(0,2), (0,1), (1,2)\}$ (Table 4).

Table 4. Two-dimensional coordinate grid of sizes $m_1 \times m_2 = 2 \times 3$ based on the IRB $\{(0,2), (0,1), (1,2)\}$

No	Forming coordinate grid $m_1 \times m_2 = 2 \times 3$			
	Node point	Summation (mod2, 3) of the IRB $\{(0,2), (0,1), (1,2)\}$		
		(0,2)	(0,1)	(1,2)
1	(0,0)	+	+	-
2	(0,1)	-	+	-
3	(0,2)	+	-	-
4	(1,0)	-	+	+
5	(1,1)	+	-	+
6	(1,2)	-	-	+

So long as the terms $(0,2)$, $(0,1)$, $(1,2)$ of the three-stage ($n=3$) sequence themselves are two-dimensional vector sums also, the set of the modular vector sums ($m_1=2, m_2=3$) forms a set of nodal points of annular reference grid over 2×3 exactly once ($R=1$). If the set of all nodal dots of the two-dimensional ($t=2$) coordinate grid corresponds one-to-one to a set of vector data we call this intelligent coordinate system. For example, the first of two ($m_1=2$) number indicates an attribute, while the second – three ($m_2=3$) categories of the attributes

by the same vector data. Hence, only three ($n=3$) two-stage ($t=2$) sub-sequences of the three-stage ($n=3$) sequence allows generate six ($2 \cdot 3=6$) distinct data sets, each of them contains information about both the number of attribute (first digit) and category of this attribute (2-nd digit) at the same time. Hence, the IRB $\{(0,2),(0,1),(1,2)\}$ forms both the two-dimensional coordinate system 2×3 over torus and two-dimensional vector data coding system under the coordinate system (Table 5).

Table 5. Two-dimensional vector data coding system based on the IRB $\{(0,2),(0,1),(1,2)\}$

No	2D vector data coding system $m_1 \times m_2 = 2 \times 3$			
	Vector data	Vector data combinations		
		(0,2)	(0,1)	(1,2)
1	(0,0)	1	1	0
2	(0,1)	0	1	0
3	(0,2)	1	0	0
4	(1,0)	0	1	1
5	(1,1)	1	0	1
6	(1,2)	0	0	1

These techniques make it possible forming a new class of intelligent two- and multidimensional vector data coding and processing generative for enhancement a priori no infinite big vector data information contents under intelligent manifold coordinate system.

4. Characteristics of Intelligent Manifold Coordinate Systems

A wide range of characteristics of intelligent manifold coordinate systems based on 2D and 3D IRB given in the Table 6.

Table 6. Characteristics of intelligent coordinate and vector data coding systems based on 2D and 3D IRB, $n \leq 7, R=1$

n	Cardinal number of vector IRB		Sizes of space grid	
	2D	3D	$m_1 \times m_2$	$m_1 \times m_2 \times m_3$
2	1	-	1×2	-
3	4	-	2×3	-
4	24	-	3×4	-
5	272	-	$4 \times 5,$ 3×7	-
6	256	128	$5 \times 6,$ 3×10	$2 \times 3 \times 5$
7	360	180	$6 \times 7,$ 3×14	$2 \times 3 \times 6$

One can see that cardinal numbers of tabled two- and three-dimensional intelligent manifold coordinate system are increasing with rising number n of basic vectors of the system. There are a large class of multidimensional intelligent coordinate and vector data encoding systems with parameters $n, m_i (i=1,2,\dots, t), S$ and $n(n-1) \leq S < n(n-1)(n-1)$. It is known that cardinality set of optimum multi-modular ring monolithic vector code increases out many times in increasing number n of 1-modular code with $n=168$ have 4676 distinct its variants. There are 360 distinct variants of 2-modular

optimum relationships of order 7, as well as, 180 distinct variants of 3-modular ones.

A chart of manifold coordinate system $m_1 \times m_2 \times \dots \times m_t$ about t concurrent ring axes m_1, m_2, \dots, m_t with common ground “+” shows in Fig.4.

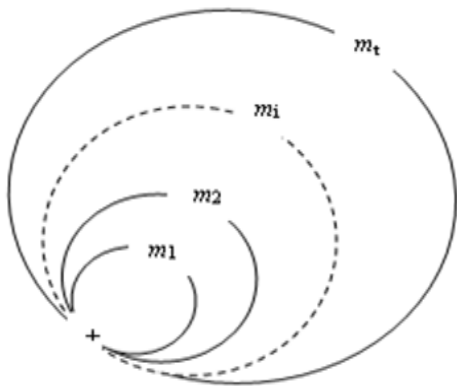


Fig.4 A chart of t -dimensional manifold coordinate system $m_1 \times m_2 \times \dots \times m_t$ about t concurrent ring axes with common reference point “+”

In the figure we see illustration a planar projections of spatially disjoint axes m_1, m_2, \dots, m_t of t -dimensional manifold reference grid $m_1 \times m_2 \times \dots \times m_t$.

Detailed description of vector data combinatorial configurations design under intelligent manifold coordinate systems, using their reconstruction and topological transformations as spatial cyclic groups of 2D and 3D IRB for optimization of vector encoding systems with examples at the practical level given

in [19], [20].

Next, we consider a new type of configurations among the most perfect vector Ideal Ring Bundles, which properties hold for the same set of the rings in varieties permutations of terms in the IRBs, namely doubled IRBs. For example, 43-fold ($S=43$) rotational symmetry makes it possible to generate under the intelligent torus coordinate system $m_1 \times m_2 = 6 \times 7$ elegant ensemble of eight two-dimensional seven-stages ($n=7$) doubled IRRs. One of them depicted here (1,1)

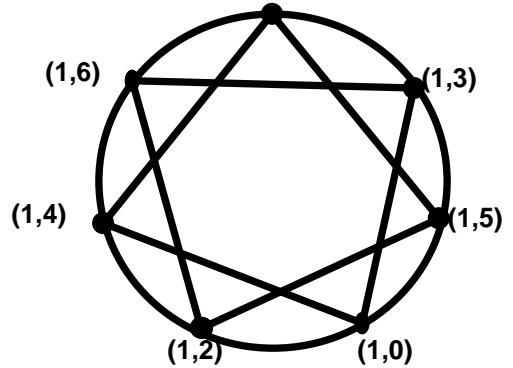


Fig. 5 Graphic representation of doubled IRB generated under the intelligent torus coordinate system $m_1 \times m_2 = 6 \times 7$

To see this we can observe set of two 2D seven-stages ($n=7$) IRBs: $\{(1,1), (1,3), (1,5), (1,0), (1,2), (1,4), (1,6)\}$ (ring cycle), and $\{(1,1), (1,5), (1,2), (1,6), (1,3), (1,0), (1,4)\}$ (star cycle). Each of them provides generation of intelligent coordinate system. We have found numerous ensembles of the sets with differing kind of S -fold spatial symmetry. The more n the more various remarkable properties discover grandiose picture of spatial harmonious about the intelligent manifolds. That's why we can evident

once more on everlasting global worldwide harmony.

"...harmony that the human mind is editable to reveal in nature is the only objective reality, the only truth we can achieve; and what I will add is that the universal harmony of the world is the source of all beauty, it will be clear how we should appreciate those slow and difficult steps forward that little by little open it to us..."

Jules Henri Poincaré

5. Conclusion and Outlook

The intelligent manifold t -dimensional coordinate systems based on the Ideal Ring Bundles provide, essentially, a new concept for vector data coding, processing, computing, and systematic researches originated on idea of minimizing vector basis of the intelligent coordinate system, using vector combining summation over the basis to cover the coordinate system reference grid. Moreover, the optimization has been embedded in the underlying configurations. The remarkable properties and structural perfection of two- and multidimensional IRBs provide an ability to reproduce the maximum number of combinatorial varieties in the system with limited number of vectors. These researches involves techniques for improving the quality indices of engineering devices or systems with respect to big vector data processing and computing speed, data transfer rate, information redundancy and code security communication. Vector data processing under the optimized manifold coordinate systems and vector data codes

provide competitive advantages of the vector information technologies with respect to processing speed and storage capacity due to data coding of compound attributes for needed their number and categories simultaneously. The creative qualities of the combinatorial structures allow classify them among intelligent information systems [18,19]. Study the properties allows a better understanding of the role of geometric structure in the behaviour of artificial and natural objects in different dimensionalities. At last developing a new vision of remarkable conjunction both rotational symmetry and asymmetry as real existing perfection of the world discovers direct application the underlying scientific approach for better understanding role of natural space geometric laws for development perspective R&S projects in contemporary vector information technologies, computing, systems engineering, and education. We can notice the fundamental role of the laws of real space as storage medium information about grandiose harmony of the Universe for generative design. It's just what the "intelligent" rotational symmetry and asymmetry provide mutual penetration existing eternal world intelligence of the Universe.

The underlying skills are useful at high schools and universities for in-depth training of students, which study computer sciences and information technologies, involving contemporary combinatorial and algebraic theory for increasing interest to scientific researches.

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