# Putting the Rhythm in Algorithm: Composing rock drum kit solos using stochastic processes

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## *Abstract*

While the drum kit is not usually thought of as a solo instrument, there is a substantial literature of solo works for the kit. Many of these were composed for pedagogical purposes, but composers including John Cage and Frank Zappa have written "serious" solo compositions for the kit. This project investigates the composition of solo drum kit pieces using random processes. Unlike in Cage's oeuvre, the goal is to produce pieces consistent with a 4/4 rock idiom, including "keeping time" with bass, snare, and cymbal, and "linear fills" with bass, snare, and tom-toms. In addition, we impose constraints intended to keep the music playable by a moderately experienced drummer at a reasonable tempo. These constraints include avoiding the most awkward hand crossings and limiting the number of consecutive strokes that need to be played by the same hand or foot.

## 1. Rock Drumming

Many authors have explored the connections between mathematics and rhythmic drumming, but most mathematical analysis has focused on rhythms played on one drum with one hand. A few authors have looked at multiple drum lines [1] and drumming with both hands [2, Chaps. 29 and 32]. There seems to have been little mathematical exploration of drumming on a full rock drum kit, however. The author's previous work [3] involved the use of matrices to count the number of rock drum fills with certain conditions. Subsequent work by the author [4], [5] on using Markov chains to generate weaving patterns inspired the idea of modifying the matrices used in counting to generate patterns instead.

The rock drum kit shown in Figure 1 is a typical example of a "five-piece" kit used by a right-handed player. The five pieces refer to the snare drum, high tom, bass drum, mid tom, and low tom, as labelled in Figure 2. The three cymbals shown (high hat, crash, and ride) are also fairly typical. Note that the foot pedal can be used to open and close the high hat, for a variety of different sounds. The foot pedal can also be used to play the high hat by closing it rapidly, rather than by striking it with a drumstick. The ride cymbal can also produce a variety of sounds by striking it in different places, such as the edge and the bell (the area

around the center). While the crash cymbal is common in many types of music, it is not used in the pieces discussed in this paper.

*Figure 1: Platin Drums Classic Set 2216 Amber Fade [Wikipedia user "Mark dolby"]*

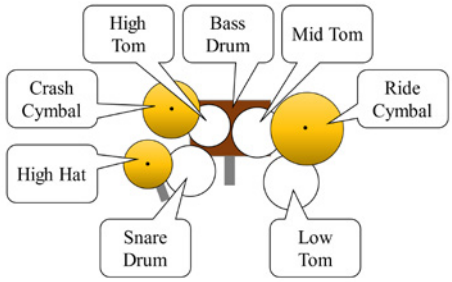


*Figure 2: Layout of a typical (right-handed) five-piece kit*

Staff notation for drums uses a standard musical staff and notes, with the lines and spaces repurposed to indicate the different drums and cymbals rather than actual pitches. One system commonly used puts cymbals on the top line and above, bass drum on the lowest space, and the other four drums in between, as indicated by the abbreviations on the left side of Figure 3. It is also usual for the note heads to be replaced by × shapes for cymbals, as shown in the figure.

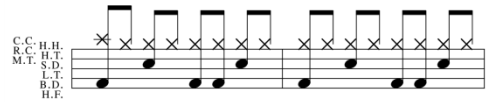


*Figure 3:* From "Are You Gonna Go My Way", Lenny Kravitz

There are basically two different kinds of patterns in rock drumming, "playing time" and "filling". When playing time, the drummer plays a repeated pattern, as shown in Figure 3, which provides a background for the other instruments and vocals. When filling, as shown in the last part of Figure 4, the drummer plays a "mini-solo", usually during a break in the melody carried by the lead singer or instrumentalist.

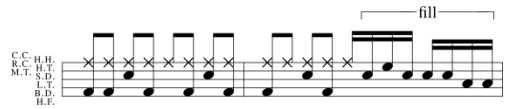


*Figure 4: From "Crosstown Traffic", Jimi Hendrix*

## 1.1 Playing Time

Playing time generally involves three categories of instruments: cymbals, abbreviated C.C., H.H., R.C., and H.F. in Figure 3 and played with the right hand (and left foot in the case of H.F.), a backbeat played on the snare drum (S.D. in Figure 3) with the left hand, and a bass line, which is anchored by the bass drum (B.D. in Figure 3), played with the right foot, but may also involve the snare and toms played with the left hand. Our algorithms will generate each of these lines separately and then put them together.

Cymbals generally play a steady rhythm throughout a section of time, using one or two different cymbal sounds. The algorithms choose a pattern and a set of sounds separately and combine them. In order to make each combination of rhythm and sound idiomatic for a given style (or sometimes just to make it playable), the algorithm chooses from a library of different cymbal lines. Each line in the library combines a pattern with a set of sounds, tweaked as necessary for the desired style.

In addition to different styles of popular music, musicians often refer to different rhythmic "feels" such as 8th note, 16th note, half-time, straight, swung, and so on. In many cases, these feels affect the placement of notes in different subdivisions of the beat. Because of this, there are slightly different versions of the algorithm for some of the styles and feels, notably straight 8th, straight 16th, shuffle, half-time shuffle, and "Latin".

## 1.2 Filling

Most versions of the algorithm use "linear fills", with certain extra conditions imposed. "Linear" means the notes are played strictly one at a time, as in the fill section of Figure 4. In addition:

1. Only the snare drum, the three toms, the bass drum, and rests will be used in the fill, as shown for example in Figure 4.
2. We will always use "right lead", meaning that every alternate subdivision of the beat, starting with the first, will use the right hand. Any skipped subdivisions (rests) will also skip the corresponding hand, as shown in Figure 5.
3. Certain "difficult" transitions will be avoided.
4. The bass drum will not be played three times in a row without a rest.

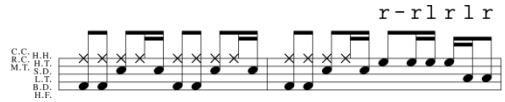


*Figure 5: From "Come Together", The Beatles*

To be specific about the difficult transitions, note that in Figure 1 the high and mid toms are at about the same height off the floor, and the snare and low tom are at about the same height. (The use of "high" and "low" in the name of the toms indicates pitch rather than spatial placement.) This makes the following two sets of transitions unusually difficult, though not impossible: right hand on snare to left hand on low tom or left hand on low tom to right hand on snare, and similarly for right hand on high tom and left hand on mid tom. In each of these cases, playing quickly requires the player's arms to cross in a way that makes it difficult for the arms to avoid hitting each other. Similarly, playing the bass drum three times in a row quickly is a fairly difficult task for an intermediate player.

The rhythm of the fill also varies somewhat according to the style and feel of the music. The straight feels use 16th note subdivisions as the basic unit, while the shuffle feels use triplets. The "Latin" style required a larger change, since linear fills are not particularly idiomatic for this style at all. Instead, the algorithm uses a unison fill where both hands (or one hand and one foot) play at the same time for a series of notes on the same pair of drums. The choice of drums is generated similarly to how a linear fill would be without conditions 2-4, and the placement of the drums is generated

similarly to the base line as described in Section 1.1.

## 2. The Algorithms

The author's algorithms use Markov chains to control the various aspects of the drum composition. The use of this technique in artistic analysis goes all the way back to Markov himself, who used it to investigate letter patterns in the works of Pushkin [6], [7]. A Markov chain is a mathematical model where a random process passes through a series of states, with the probability of passing to each next state depending only on the previous state attained. For example, there are twelve states corresponding to the dynamics of each section of the music: *pianissimo* and decreasing, *pianissimo* and increasing, *piano* and decreasing, *piano* and increasing, and so on for *mezzo piano*, *mezzo forte*, *forte*, and *fortissimo*. The probability of each dynamic level in a new section of music depends only on the level in the previous section. These probabilities are arranged with the goal of producing an overall effect which is dramatic without being jarring.

The compositional structure of the pieces produced is fairly conventional for a drum solo written for pedagogical purposes. The piece is divided into four sections, each of which consists of three measures of time and a measure of fill, as depicted in Figure 6. Each section is played twice, the second time incorporating an increase or decrease in dynamic level during the end of the last measure. This change in the dynamics may lead either toward or away from the dynamic level of the next section, providing either gradual transition or dramatic contrast.

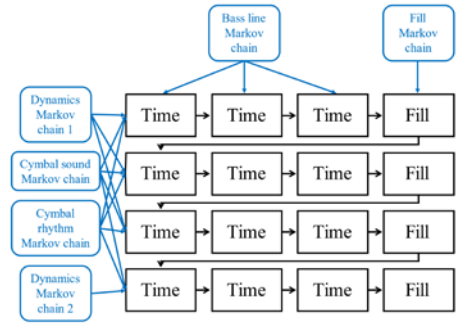


*Figure 6: Overview of the Markov chain scheme*

Markov chains are used on both the macro and micro scales. On the macro scale, one chain controls the dynamics of each section, and two more control the cymbal choice and cymbal pattern. Within each section, a Markov chain controls the micro-scale rhythm of the bass line and another controls the composition of the fill which ends the section. Rather than treating each bass note individually, the algorithm groups them into pairs or triples, as appropriate for the rhythm of the piece. This allows the possibility of emphasising patterns characteristic of certain genres, such as hard rock, funk, and "Latin". It also allows us to forbid requiring any hand or foot to be used on three consecutive subdivisions of the beat. This scheme is illustrated in Figure 6.

## 3. Transition Matrices

The probabilities used in the Markov chain are given as input parameters to the algorithm in the form of matrices, known to mathematicians as transition matrices [8, p. 6]. The entry of a transition matrix in the $i$-th column and $j$-th row is the probability of transitioning to state $j$ given that the system is in state $i$. An example is shown in Figure 7. For each matrix an initial vector is also

specified, giving the probabilities of each possible starting state.

$$\begin{array}{c} \\ HH \\ RC \\ Bell \\ HF \end{array} \begin{array}{c} \begin{array}{cccc} HH & RC & Bell & HF \end{array} \\ \begin{pmatrix} 0.1 & 0.4 & 0.4 & 0.4 \\ 0.6 & 0.1 & 0.4 & 0.4 \\ 0.2 & 0.4 & 0.1 & 0.2 \\ 0.1 & 0.1 & 0.1 & 0 \end{pmatrix} \end{array}$$

*Figure 7: Transition matrix for cymbal sounds in the funk style. "Bell" indicates the ride cymbal played on the bell rather than nearer the edge.*

In some cases, it was found to be desirable to use two different transition matrices. For the fill sections, one matrix was used for transitions from the right hand to the left hand and a different matrix was used for the opposite direction. This allowed better control of forbidding difficult transitions. For dynamics, the final section of the piece uses a different matrix than the others in order to provide a more emotionally satisfying finish.

The entries in these matrices and vectors were chosen by the author to reflect the aesthetics of particular styles, as mentioned above. One interesting by-product of using matrices is that a linear combination of transition matrices is still a transition matrix if the weights are chosen appropriately. This allows the user of the algorithms to select a style that is, for instance, "one-third of the way" from hard rock towards funk.

## 4. The Computer Program

The author has written a set of computer programs [9] in the Julia programming language to generate random drum solo compositions according to the algorithms defined above. Since it was desired to produce pieces that could be played either by humans or computers, the programs output abc notation [10] in the form of a very portable text file. The abcm2ps software [11] can then be used to produce musical scores, whereas the abc2midi software [12] can be used to produce a MIDI file which can be played by most computers and many electronic instruments. Both pieces of software are free and open source (FOSS) and have versions available for Windows, MacOS, and Linux.

## 5. Conclusion and Future Work

The combination of the parameter decisions made by the user and the randomness generated by the computer produces music which is recognizably from the rock idiom but distinct from the composition style of any existing human composer, including the user. While more in the category of pedagogical etudes than avant-garde compositions such as those of Cage [13] or Zappa [14], pieces produced by the algorithms have been played for a number of small but appreciative audiences. The goal of this project is to push the boundaries of musical genres and challenge listeners to think about how we define and categorize music.

Programs and parameters have so far been generated for the styles of hard rock, funk, and "Latin", and the feels of straight 8$^{th}$ notes, straight 16$^{th}$ notes, and half-time shuffle. Future work should include refinement of the Latin style, especially the bass line and the instrumentation, and the implementation of more styles, which could include blues, country, disco, reggae, and jazz styles including swing. More experimentation with interpolating between styles is also called for. Shuffle, waltz, 6/8, and 12/8 feels would be good additions as well.

Work has started on the shuffle feel, but the appropriate style calls for a wider range of instrumentation than currently implemented. In particular, the bass line

should be augmented with “ghost” notes played softly on the snare drum. For other styles, base line notes on the low tom should be included as well. In the country style, the backbeat is often played with a technique known as a cross stick, whereas in the blues style it might be played with a rim shot. More cymbals could be added, including crash, splash, and “china”. Some blues, rock, and Latin styles also play cymbal-type patterns on the snare drum, toms, and other instruments, including wood blocks and cowbells, in addition to the actual cymbals. (Ringo Starr played patterns on the low tom in a number of classic Beatles tracks, including “Come Together” and “Sgt. Pepper’s Lonely Hearts Club Band”.)

Finally, there are many drum techniques which have not yet been incorporated into the algorithms. In addition to ghost notes, other variations in accent frequently occur in drum music, especially in fills. Fills can also mix the use of 16$^{th}$ notes and triplets, as well as mixing linear and unison patterns. It is not necessary to always use right lead, and in fact “breaking” it is an important feature of some techniques including rolls, flams, cymbal chokes, and double- and triple-strokes. All of these techniques and styles are accessible to the intermediate drummer, so there is still quite a ways to go toward a complete algorithm for generating drum kit etudes.

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