A visual interpreter for pre-defined muqarnas units

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Summary

The idea of designing muqarnas from pre-defined units opens the way for parametric design. Parameterization is the beginning of the creation of a language in describing muqarnas. My final goal is to deliver an interpreter that facilitates thought and speech in terms of squares, rhombi, almonds, bipeds, candles, etc… Its software reads abbreviated language and produces what we have in mind: 2D plans and 3D visualizations. In the end, it will generate modular muqarnas art!
After a cultural introduction and a short overview of research on pre-defined muqarnas units, the concepts behind the parametric toolbox and the interpreter will be presented.

Cultural Identity of muqarnas

Muqarnas is considered a typical Islamic architectural element. As a sculptural ornament, it decorates the transition zones between the dome and the square or rectangular building plan. One may find muqarnas above entrance doors, in niches, minaret balconies, fountain bassins, iwans; more often in religious buildings but also in secular buildings such as palaces, merchant houses, and caravanserai. According to Islamic artisans, the entire structure has a religious meaning. The centre of the uppermost star points to unity and the planes of the muqarnas cells, in all their different sizes and orientations reflect multiplicity. The Light shines on every plane, directly or indirectly, resulting in a play of light and shadow. They say that muqarnas stand for unity in multiplicity. According to Turkish architects, the acoustical function is important in the corners of the building and in the top of the mihrab. Depending upon the selected materials, it absorbs sounds or amplifies the human voice.
Over time, different materials have been used, primarily brick, tiles, plaster, stone, and marble. Regional styles do exist. The distinguishing features of a Spanish muqarnas are very different from an Iranian, Seljuk, Armenian, or an Ottoman muqarnas.

Figure 1, Niche in Ilkhanid Shrine Complex in Natanz, © Margi Lake

Figure 2, Iwan in Ilkhanid Shrine Complex in Natanz, © Margi Lake

Research on pre-defined muqarnas units

Alacem [1] , Garofalo [2], Harmsen [3], Kazempour [4], Sakkal [7], Sarvdalir [8], and Yaghan [10] promoted the concept of pre-defined muqarnas units. They all underline a modular approach. My research aims to develop software for an interpreter, part of a toolbox that supports the analysis and visualization of muqarnas consisting of pre-defined units.

Motivation

This research has been supported by Anissa Foukalne, a Dutch art historian and museum project coordinator. She encouraged me to develop a hands-on educational workshop on muqarnas for teenagers. This approach is designed to prompt teenagers to use their brains and hands. Six boxes have been filled with paper tiles to create 2D plans like jigsaw puzzles. Six boxes have been filled with hundreds of Lego-like 3D printed units to assemble 3D muqarnas. A short edition of this workshop takes one hour. Children learn about the cultural background of muqarnas and how to implement the design principles. They decode 2D plans and create a simple 3D muqarnas. An extended edition can be done in a one-day workshop. The result will be a replica of an existing four-level muqarnas as shown in figure 3.

Figure 3: example of a four-layer muqarnas built with 3D printed units.

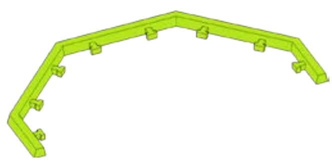


Figure 4: a construction plate of the above mentioned muqarnas.

Toolbox

Over the years, my toolbox grew larger and larger. It started with the 2D paper tiles. They are still helpful for quick analysis and prototyping. The next step was 3D printing of Lego-like building blocks using SketchUp. A 3D assembly of these building blocks provides a much better representation than any computer animation will do on a screen. For students, a GeoGebra application has been developed to create 2D plans and 3D models on the computer through drag-and-drop. The main advantage is that they can save their work on the internet to show the results to friends and at home.

All these activities marked the beginning of the learning cycle. The next task was to take advantage of parametric design tools like Rhino / Grasshopper. From this came the idea to develop an interpreter as an interface between abbreviated language and GeoGebra or Rhino / Grasshopper.

A systematic approach

In general, there are three main types of muqarnas system: the 30° system, the 45° system and the pole system. The 45° muqarnas system, also called the square system, is popular among researchers because it has a limited number of pre-defined units. Every unit is symmetrical, all angles are multiples of 22,5° and connecting sides have the same length. Furthermore, all units have the same height. Therefore, a unit has only three parameters: its class being full or intermediate, the angle at the front, and the angle at the back. Each unit has been assigned a name and an abbreviation, like full square (A) or intermediate rhombus (H). Therefore eight times full half sharp rhombus (M), make up eight times 45°, a complete eight-pointed star. Usually, these eight (M) form a top layer dome and the underlying layer could be a sequence of full jug (D) and intermediate goose feet (N). Figure 5 shows a visual presentation of a subset of these pre-defined units. A 2D plan with these two top layers is shown in figure 6. Figure 7 shows an artist's impression generated by Rhino.

In the example of figure 6 and 7, the pink top layer consists of an eight-pointed star (M), the layer underneath consists of jug (D) and goose feet (N), the third layer of candle (J), almond (C), star (M), fourth of (I) (D) and (N) etc...

Interpreter

The next development was to describe muqarnas with abbreviations and have an interpreter do the translation of these abbreviations into 2D plans (squares and triangles) and 3D objects (solids of planar and curved surfaces). This complicated matters because in half domes, mihrabs, and portals you only need the left or right part of a unit to close the boundary. Furthermore, additional rotations and translations are required as well as increased or reduced scales, so the interpreter had to be intelligent. It had to comprehend the concepts of repetition and reflection. In case a layer is made up of eight (M), the input should be something like "eight times (M)", instead of (M M M M M M M M) and when it is "four times (I D N D I)", the input should not be (I D N D I I D N D I I D N D I I D N D I), but something like "(I D N), reflect and repeat again and again". Why code whole layers, when the 45° generating part is only one eighth of the whole 360°? Why code substrings again and again? The interpreter should allow for grouping into named substrings. For example, sometimes, the underlying layer closely resembles the parent layer. The interpreter should understand recursion in terms of "this layer is the previous layer plus something extra". So, the interpreter should know that the reflection of the left side of a unit is the right side of that unit and that the reflection of a 45° turn to the right is a 45° turn to the left and vice versa. The aim is to specify muqarnas as short as possible.

Pre-defined units

Harmsen recognized twelve pre-defined units. The left columns in figure 5 show these twelve units, the column on the right shows two more units, the already mentioned star (M) and goose feet (N), which are very common in Seljuk, Armenian, and Ottoman muqarnas. While working on new examples, more units were discovered. The number of permutations in the 22,5° / 45° system is however limited to maximal 42. This mathematical analysis predicted that there are more units than the fourteen mentioned in figure 5. A number of these were discovered in the Istanbul Marmara Theology Faculty Mosque. They will be discussed later. Further research is needed to discover examples of the remaining rare units.

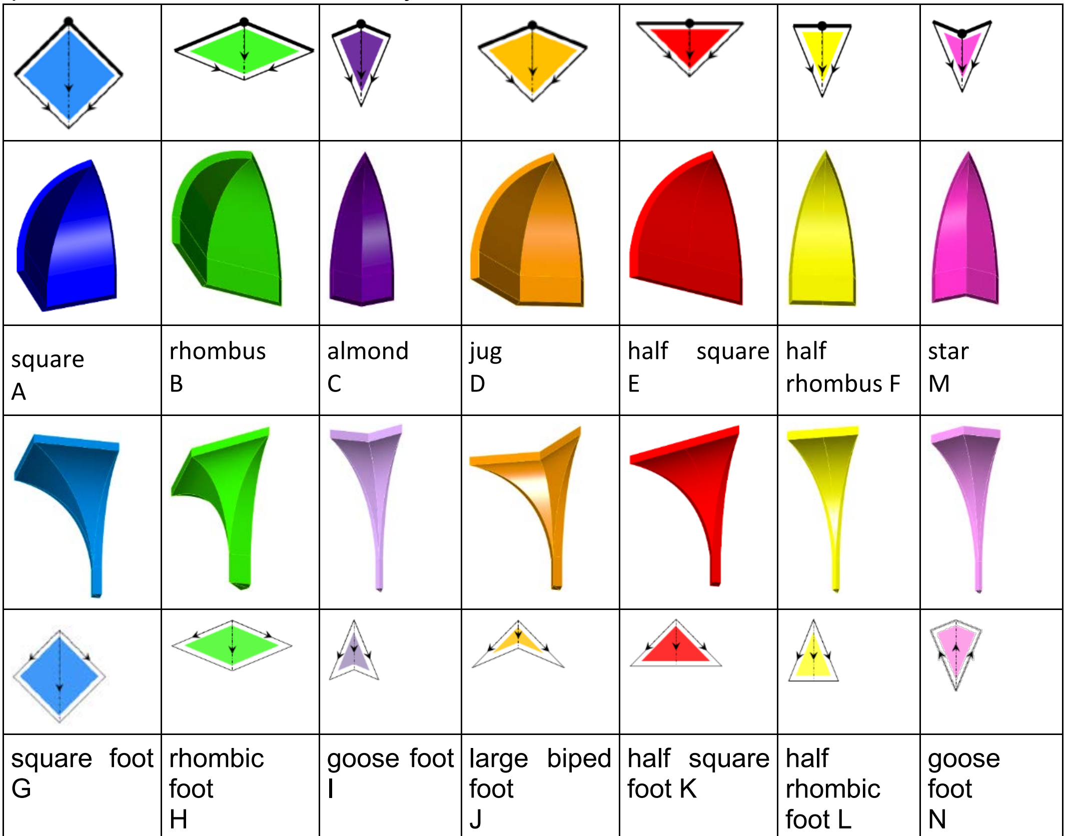


Figure 5: basic overview of the pre-defined units.

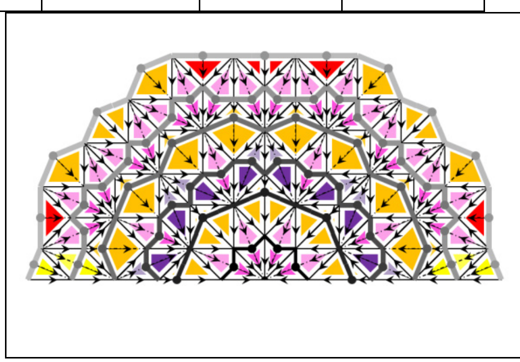


Figure 6: A 2D plan

Figure 7: the corresponding 3D visualization

Implementation

The interpreter is written as a parametric Microsoft Excel visual basic application due to its capacity to organize output in worksheets. One worksheet contains a table with the definitions of all the pre-defined units, the other worksheets contain the muqarnas specifications. The interpreter searches through the table for the characteristics of each object specified in the input string.

As mentioned before, the characteristics of a unit are the angle at the front, the angle at the back, the type (full or intermediate) and the position of the foot column. Other parameters are the design of the interior and its colour. The Excel application has many functions, such as computing the position of each muqarnas unit, drawing a 2D plan, creating input file for GeoGebra, creating input file for Rhino / Grasshopper, generating all kinds of statistics, maintaining a where-used list by item and by layer, reporting errors and warnings on design issues, etc... Even a debugger is available to facilitate prototyping with a step-by-step drawing mode.

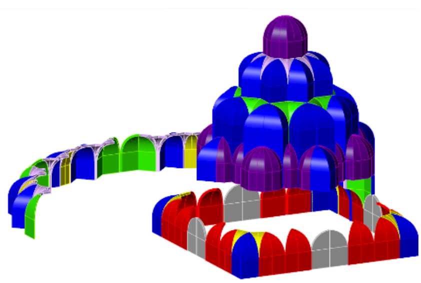


Figure 8: a debugger helps to track design errors

Example design Erzurum

The design of the muqarnas dome of Erzurum is very straightforward. It has only a few muqarnas units. The interpreter passed the exam because it could define the basic pattern and understand all the required repetition, reflection, and recursion.

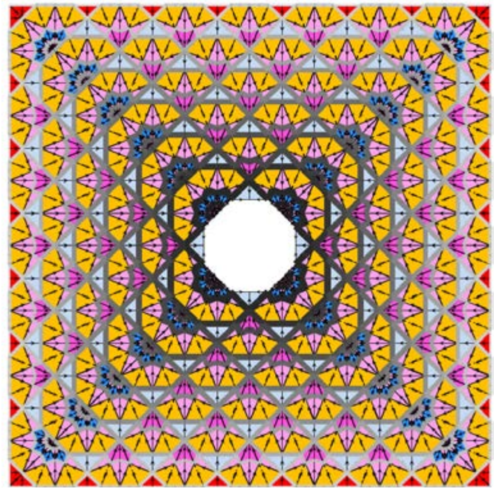


Figure 9: proposed 2D plan Erzurum

Rhino / Grasshopper

The capabilities of parameterization in Rhino / Grasshopper are unprecedented. Only a single parametric object has to be defined. The input file behaves like a set. For each item in the set, Grasshopper draws that item at the specified position with the specified height, width, orientation, and characteristics.

Depending on the input file and the parameters, Grasshopper generates a single muqarnas element, a complete half-dome, or a full dome.

In Grasshopper, each muqarnas is a true 3D object with length, width, and height. It is a closed solid: it has mass! Since the muqarnas is a solid, Rhino can export STL files for a 3D printer.

In short, the Excel application and the Rhino / Grasshopper definition can realize any 45° muqarnas plan. Both are parametrically designed. Given that it is a matter of parameters, it should be fairly simple to implement the 30° system. More of interest is how close the 45° system resembles case examples.

Rethinking Pre-defined units

Different authors have different opinions on what a pre-defined unit contains.

Rethinking the concept of pre-defined units has been crucial to integrate these opinions. Harmsen explained the concept of roof and facet with the drawings in figure 10.

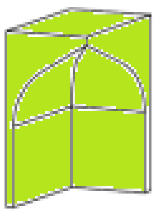
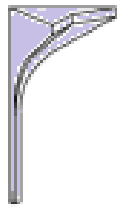
	
a full cell unit has a roof and a facet (walls)	an intermediate unit has a roof, its facet is only a small column

Figure 10: a full cell and an intermediate

Harmsen distinguishes between two concepts for the stacking of units on each other: “back on front” and “back on curve”. Figure 11 shows examples.

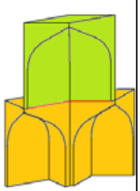
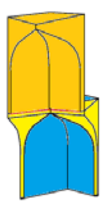
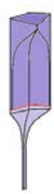
Back on curve	Back on front	Back on front
		
green full rhombus **B** sits on two orange full jugs **D**	orange full jug **D** sits on two yellow intermediate half rhombus **L** with a full blue square **A** in-between	purple full almond **C** sits on a light purple intermediate small biped **I**

Figure 11: back on curve / back on front

Although “back on front” is most dominant in Harmsen’s thesis, most muqarnas are of type “back on curve”. Let’s investigate her example of a tiny muqarnas with a full rhombus (B) on top, having its back on the curves of two preceding full jugs (D). Between the latter is not an intermediate unit but a rotation of 135°. According to Sarvdalir and Sakkal, the sides of the two bottom jugs belong to the top unit. Therefore, an interpreter should recognize that the foot of an underlying layer has to be treated together with the roof and facet of the upper layer, as in the example of figure 12.

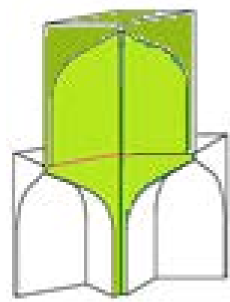


Figure 12: a full muqarnas unit has a roof, a facet, a foot, and a column

In my view, there are two kinds of full muqarnas:

- those with a roof, and a facet, sitting back on front on top of an intermediate unit
- those with a roof, a facet, a foot, and a column, its facet sitting back on curve on top of the underlying layer.

This modification of the definition opens the way to unifying the classification of authors like Sarvdalir and Sakkal with the classification of Harmsen. The top row of figure 13 shows my definitions, and the bottom row shows the matching drawings of Sarvdalir. In his drawings, a pre-defined unit spans two layers. My system of coding, like Harmsen’s, states a full unit jug (D) on the upper layer and an intermediate large biped J on the underlying layer, but in case examples, they are one: a single curved surface with one interior.

Sakkal describes Armenian muqarnas in a different way. Figure 14 matches my drawings to those of Sakkal. In his drawings, a pre-defined unit also spans two layers.

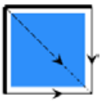
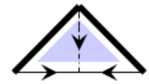
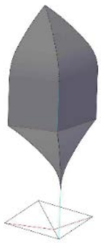
Concave	Concave	Straight	Convex	Concave	Concave
					
square 90°	large biped + jug 135°	half square 180°	225°	square 90°	half square 90°
full unit A	intermediate J + full unit D	intermediate K + full unit E	intermediate L + full unit O	full unit Oo	full unit Qa
					

Figure 13: Comparison between Harmsen and Sarvdalir

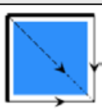
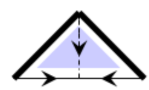
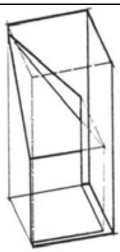
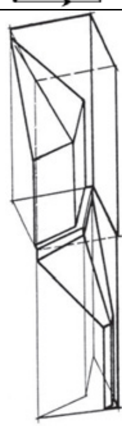
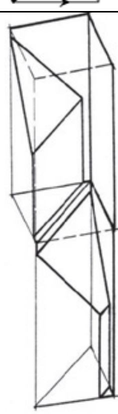
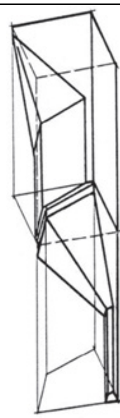
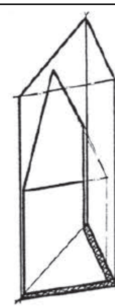
Concave	Concave	Straight	Convex	Concave	Concave
					
					
full unit A	intermediate J + full unit D	intermediate K + full unit E	intermediate L + full unit O	full unit Oo	full unit Qa

Figure 14: Comparison between Harmsen and Sakkal

Regional differences

As stated before, the differences between muqarnas in different regions or different eras are considerable and well worth further research. With the current version, it is possible to design muqarnas by applying the above-mentioned pre-defined units of the 45° system. Western Islamic muqarnas, however, have different properties. It is not possible to approximate Ottoman muqarnas by the 45° system, because, according to Senalp [9], it makes no sense to leave out important details like the thin candle, which is slimmer than the large biped (J). Furthermore, Ödekan [6] shows Turkish muqarnas with a combination of the 45° system and pentagons with rhombi of 108°.

Rare pre-defined units

Mathematical analysis predicted the existence of rare units. Figure 15 shows the possibility to square eight-pointed star units (M). The small surrounding units can be either intermediate or full elements. The Istanbul Üsküdar Marmara University Theology Faculty Mosque is a modern structure with classical Ottoman roots. The top layer of the mihrab is an eight-pointed star and the underlying layers are rectangular. At the end of the rectangle of the second from top layer, there are two squares. In between there are intermediates, but in the middle section, full units are required to close the wall of the rectangle. Figure 15 shows the surrounding wall and the positions of the foot columns.

Figure 15: Squaring an eight-pointed star in the Istanbul Üsküdar Marmara University Theology Faculty Mosque

Figure 17 shows the input for my 3D printer. It was generated directly from the Rhino STL file, whose input was generated automatically by the Excel application, without manual intervention.

Many more pre-defined units

It turned out that the Harmsen set had to be enriched with many more units to meet case examples. An investigation of all possible configurations shed light on these to-be-discovered units. The majority were found as cornerstones. Some were discovered in Armenian muqarnas, others in Seljuk. Although, the Ottoman muqarnas units appear to be similar to the Seljuk ones, they are not. For instance, an important Ottoman unit is the candle, an object with a similar function as the large biped, but much slimmer than the large biped. Its angles are no longer multiples of 22,5°.

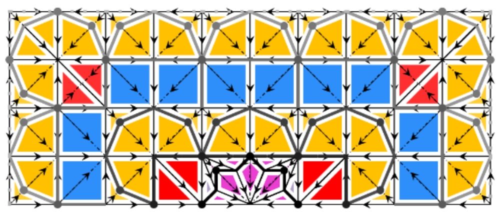


Figure 16: 2D plan of the mihrab in the Istanbul Üsküdar Marmara Mosque

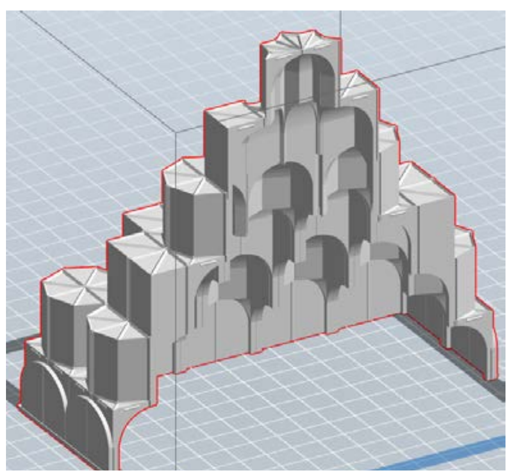


Figure 17: FlashForge 3D pre-print view of the same mihrab

Art

The next question is what should be considered art. Sometimes, the mistakes were beautiful too. These mistakes happened easily when the parameters in Rhino were not in sync with the parameters in Excel.

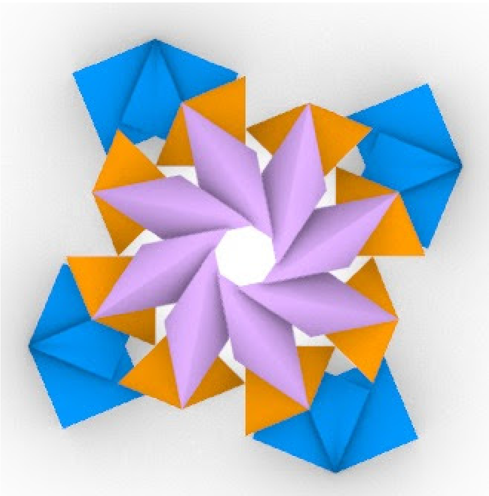


Figure 18: Parameter mistakes can be beautiful too.

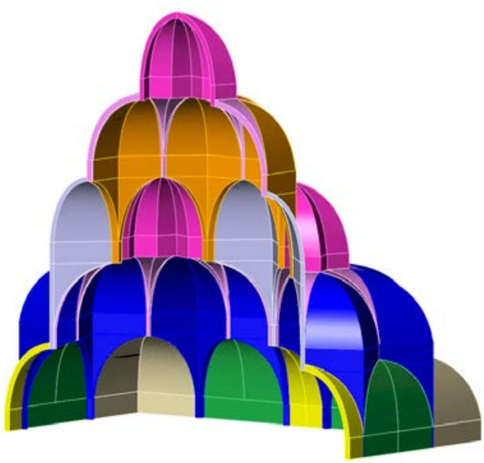


Figure 19: Rhino visualization of Portal Yeghvard St Astvatzatsin

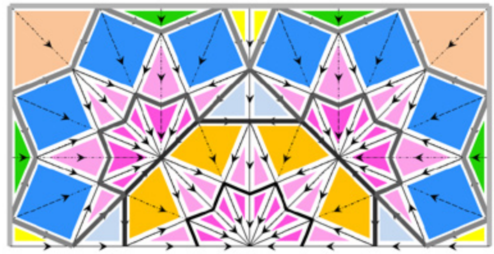


Figure 20: 2D plan of Portal Yeghvard St Astvadzatsin (source: Sakkal)

Checking 2D plans

Converting a 2D plan into a 3D visualization can be surprising. At times, it is not clear which unit belongs to which layer. For example, the southern vault of Takht-i-Sulayman has been discussed by many authors [3, 9, 10]. The 2D plan of figure 21 leaves room for different interpretations.

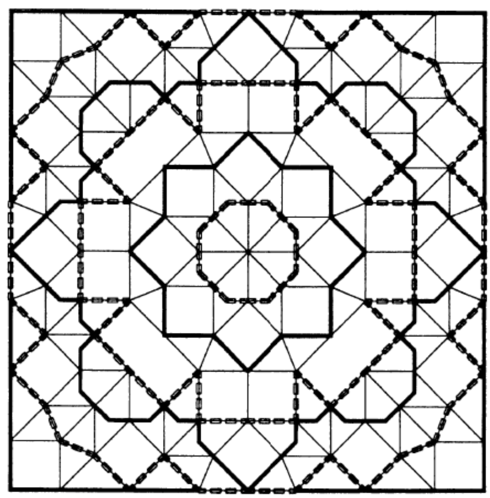


Figure 21: Reconstruction of the muqarnas of the southern vault of Takht-i-Sulayman (source: Yaghan).

In a 2D plan with less clearly defined layers, not using bold and dashed lines, there will be much more room for confusion. Therefore, I wanted visual tools and a 3D printer.

Thanks to the fast interpreter, I could instantly compare alternatives and recognize the most likely interpretation.

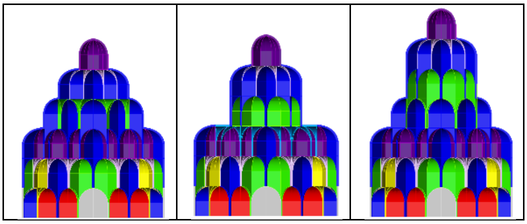


Figure 22: Three different alternatives for the southern vault of Takht-i-Sulayman.

Conclusions

The first results showed that it is possible to develop software for an interpreter for the 45° muqarnas system. Rethinking the idea of pre-defined units was a prerequisite for integrating the practitioner's view and the academic opinion. While checking 2D plans, the number of pre-defined doubled from twelve to more than thirty. More than a hundred various designs were tested and the interpreter was able to understood them all. The tool was particularly useful for quickly studying alternative variants. The interpreter is ready for the next phase.

Future Research

Much more work must be done, for example:

- Check muqarnas plans from literature with case examples
- Obtain the true measurements of more case examples
- Expand to the 30° system and eventually the pole system
- Expand to other muqarnas systems, like the Ottoman, and the Western Islamic muqarnas system
- Weekly post on Instagram
- Explore the interior design of muqarnas, for example, supporting the concept of recursion: muqarnas within muqarnas, as is found in the Armenian muqarnas system

References

My website www.fransvanschooten.nl has a webpage muqarnas.htm. It provides a long list of references and material. Also it provides examples of the input coding and the Rhino and STL files. Twice a week, I post on my account https://www.instagram.com/henk.hietbrink/

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