# Phyllotaxis Is Not Logarithmic

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***Abstract***

In the plant kingdom, with the greatest variety of forms, there is a limited number of stable patterns of arrangement of repeating elements, such as leaves, buds, seeds, scales, etc. One of these patterns is called *spiral phyllotaxis* [2]. It is widely believed that the spirals of the rows of leaves that make up this pattern are logarithmic spirals [9]. The present study refutes this view.

## 1. Phyllotaxis

In a number of common species the leaves are arranged in whorls at the level of the stem. The number *n* of leaves in a whorl varies from species to species, in the same species, and can vary in the same specimen.

Spiral patterns: *n=1*

Decussate pattern: *n=2*

Whorled = verticillate patterns: *n>=3*

Usually it is possible to distinguish two or more sets or families of spirals round the stem, which run in opposite directions and which appear to cross one another.

## 2. Rising phyllotaxis

*Rising phyllotaxis* is the phenomenon observed on plants with the spiral phyllotaxis, when the number of conspicuous parastichies is increasing from the centre to the periphery of shoot [7]. It is important to note that the pattern is not self-similar.

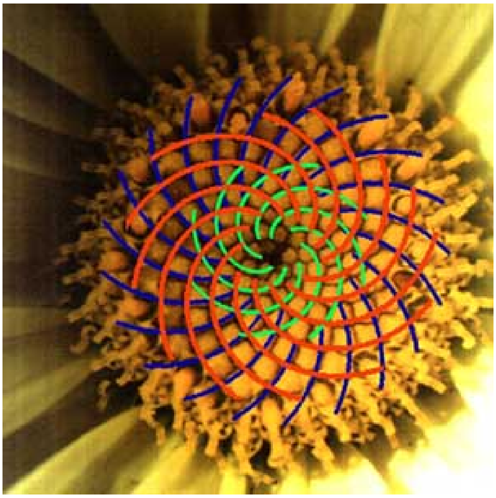

*Fig. 1. Two families of parastichies constitute parastichy pair. Several parastichy pairs can be seen on one inflorescence.*

## 3. Spirals are different

### 3.1. Logarithmic spiral or equiangular spiral

The logarithmic spiral was first described by Descartes and later investigated extensively by Jakob Bernoulli, who called it Spira mirabilis, "the marvelous spiral." Bernoulli was fascinated by one of its unique mathematical properties: the size of the spiral increases, but its shape is unaltered with each successive curve. In polar coordinates $(r,\theta)$ the curve can be written as

$$r = a\,e^{b\theta}$$

with $e$ being the base of natural logarithms, and $a$ and $b$ being arbitrary positive real constants.

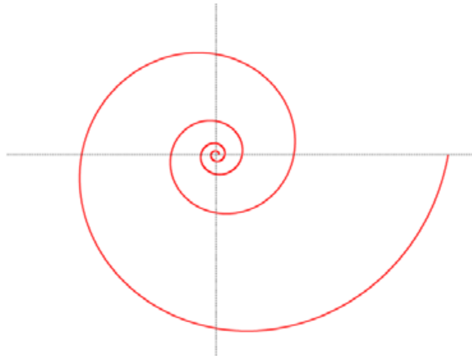

*Fig. 2. The logarithmic spiral.*

### 3.2. Archimedean spiral

It is the locus of points corresponding to the locations over time of a point moving away from a fixed point with a constant speed along a line which rotates at a constant angle. Equivalently, in polar coordinates $(r, \theta)$ it can be described by the equation

$$r = a\theta$$

with real numbers $a$ and $r$.

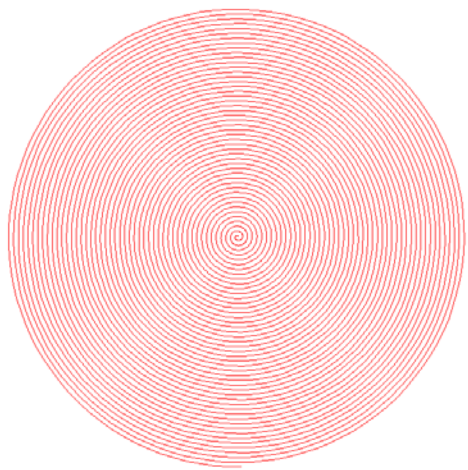

*Fig. 3. Archimedean spiral.*

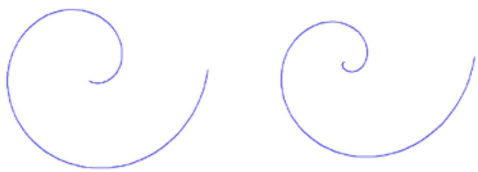

*Fig. 4. The first coil of Archimedean spiral (left) is rather similar to the logarithmic one (right) that misleads people even with trained accurate eye.*

## 3.3. Pierre de Fermat's spiral

Another possible curve is described by the equation

$$r = (\sqrt{a})\theta$$

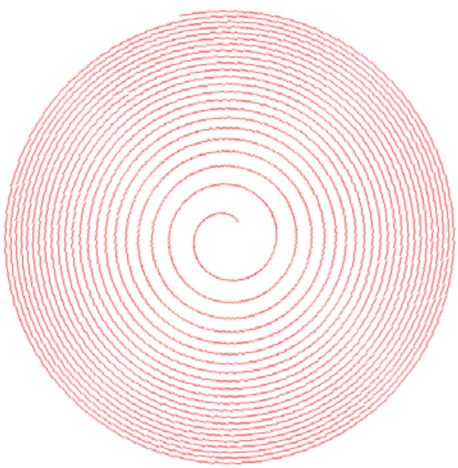

*Fig. 5. Fermat's spiral. Pierre Fermat (1601-1655): French mathematician.*

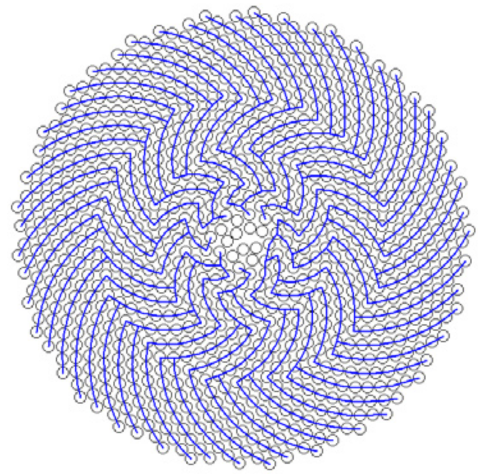

*Fig. 6. Phyllotactic pattern based on Fermat's spiral. Buds on a flat inflorescence remain the same size regardless of the distance from the center.*

## 4. Analog model

We share the opinion first expressed by Wilhelm Hofmeister in 19 century [6]. The spiral describes a pattern in which new florets emanating from the center of the flowerhead push older ones out towards the rim. Several models were put forward in which the interaction between the primordia was ascribed to the contact pressure [1], [4], [10]. Douady and Coudet demonstrated that the dynamical hypothesis put forward by Hofmeister formed the rule of an iterative system which produces the observed spiral structures. The authors showed that it was possible to obtain them in an analogous physical experiment and in a numerical simulation [5].

Our analog phyllotactic model is proposed to explain the phenomenon of *rising phyllotaxis*. The model is based on suitable analogy primordia with soap bubbles. The movement of soap bubbles is consistent with the laws of mechanics.

*Fig. 7. Analog model of phyllotaxis.*

The model uses similarity between spherical soap-bubbles-like units and primordia. The primordia arise from the liquid in the center of the cylinder top, one by one, according to the rule: every primordium *moves* in the largest available space according to the minimax principle. The primordia move radially and simultaneously, and grow in diameter until they experience contact pressure. By the way, the paths of horizontal motion of primordia are not rectilinear.

Increasing amounts of contact parastichy pairs realize due to the rearrangement of primordia during their movement from center towards the rim. Parastichy becomes contact and visible to the naked eye when primordia touch one other.

This assumption is reminiscent of Hofmeister's rule [6]. He proposed that new primordia appear periodically at the apex boundary in the largest available gap left by the preceding primordia. An important distinctive feature of our model is the formation of a pattern not only at the apex boundary, but on its entire apex area.

## 5. Mathematical description

The model is formulated in *centric representation*, where each family of parastichies is a set of identical *Archimedean spirals*. We have centric vector spiral lattice. The primordia stand in the nodes of this lattice. They are numbered according to their age that is according to the order in which they arise on the plant apex, with 0 being the youngest (Fig. 8).

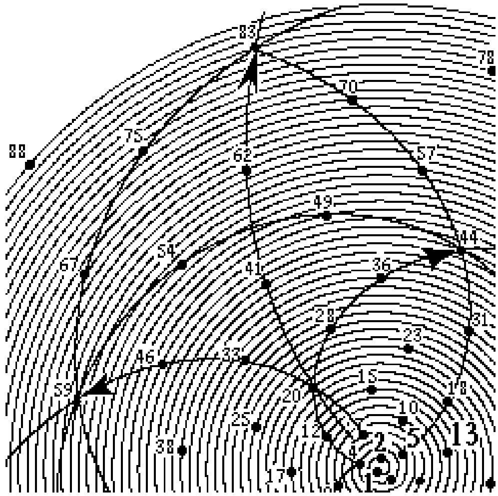

*Fig. 8. The centric vector spiral lattice.*

The numbering of primordia is in agreement with the Bravais-Bravais theorem [3]: in a family containing *n* parastichies, on any parastichy, the numbers of each consecutive primordia differ by *n*. Numbers on each *n*-parastichy are congruent *mod n*, it means, belong to the same residue class *mod n*. Each parastichy is considered as a residue class. Difference of numbers of any primordia is considered, first, as a *lattice vector*, and, second, as a *module* in its residue class.

We can add and subtract integer vectors in agreement with the parallelogram rule.

The origin and replenishment of contact parastichy is described by addition of vectors at the moment of touch of two (younger and older) primordia, moving in the opposite corners of the primitive unit cell.

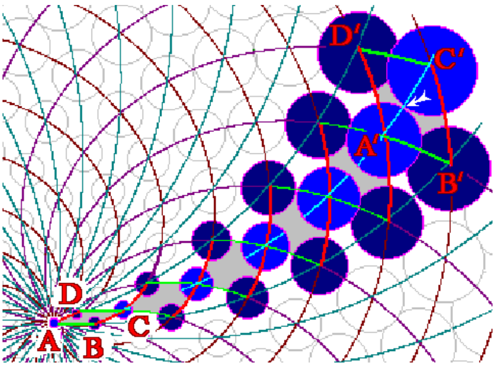


*Fig. 9. Primordia **A&C** are convergent, that means, contact parastichy appears. Primordia **D** & **B** are divergent, that means, contact parastichy disappears. Arrow show the moment of touch of two (younger **A'** and older **C'**) primordia, moving in the opposite corners of the primitive unit cell **A'B'C'D'**. At that moment new vector **A'C'** arises. Vector **A'C'** = Vector **A'B'** + Vector **B'C'***

The appearance of new vectors, or moduli, causes the appearance of new residue classes *mod m*. It is well known that there are exactly *m* distinct residue classes *mod m*, consequently, after addition of vectors *m* and *n*, *(m+n)* residue classes *mod (m+n)* will appear, this means *(m+n)* contact parastichies. A contact parastichy pair *(m, n)* is replaced by contact parastichy pair *(n, m+n)*. Thereby, Fibonacci sequence arises. As it was proved, *the rising of spiral phyllotaxis is isomorphic to increasing of Fibonacci sequence*.

Separation and divergence of primordia result in the disappearance of contact parastichy. In each contact parastichy, addition occurs among the younger primordia nearest to the centre, but subtraction occurs among the older primordia farthest from the centre. The primordia move from the centre to the rim, but location of areas of contact parastichy pairs is not changed.

The appearance of Fibonacci-type sequences is explained by misleading of primordia at initial stages of apex development.

## 6. The work purpose

In this work, we assert and prove that the *Logarithmic spiral* is not a good stencil for the phyllotactic pattern.

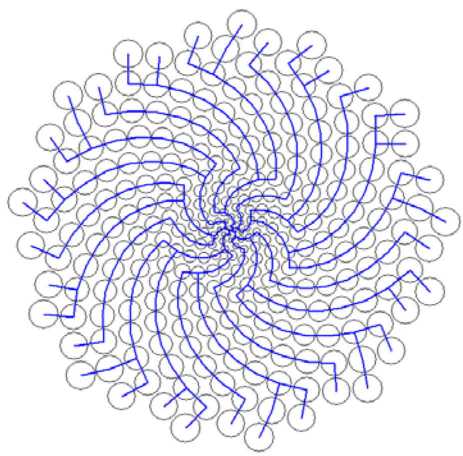

*Fig. 10. Phyllotactic pattern based on Archimedean spiral.*

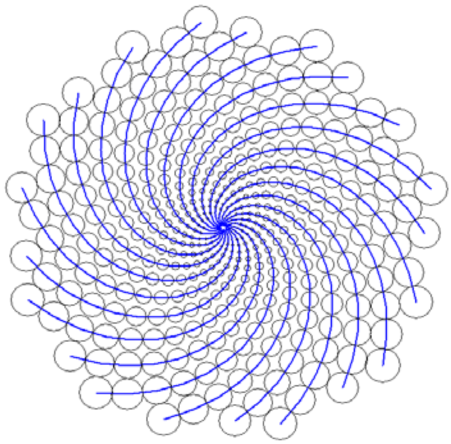

*Fig. 11 Phyllotactic pattern based on Logarithmic spiral.*

## 7. Proof

The proof of the statement is derived by *Modus tollens.*

*If an inflorescence is built by the Logarithmic spiral, then it is self-similar;*

*The inflorescence is not the self-similar (because of the rising phyllotaxis is observed);*

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*The logarithmic spiral is not involved in inflorescence formation.*

## 8. Precedents

The purposeful researches on mathematical nature of phyllotactic curves was carried out erlier [8].

A quote: " Digitized coordinates for 39 sunflowers have been analysed using power low ($r=An^k$) and logarithmic ($r=A'K^n$) spiral radial functions. The power law accurately accounts for *Helianthus tuberosus*, whereas the much larger *H. annuus* are better fitted by a combination of both types of spiral ".

Noting the great scientific value of this work, we consider it necessary to make an important comment: The authors, in our opinion, incorrectly explain the phenomenon of rising phyllotaxis.

A quote: "We refer here to a physical degradation of the seeds, which were not all fertilized and so did not mature normally; an effect which is coupled with the phenomena of rising phyllotaxis".

It is necessary to object, that rising phyllotaxis is not casual coincidence of circumstances, but a necessary and basic condition of formation of the spiral patterns.

## 9. Different objects - different spirals

In popular science literature on the formation of living nature objects, spiral phyllotaxis is often considered together with spiral objects of fauna - seashells, horns of ungulates, etc.

Let us point out the main, in our opinion, difference in the formation of the shell of the mollusk (say, nautilus) and the pattern of the plant (say, the head of a sunflower). The shell grows with its outer part, i.e. the edge of the hole from which the leg of the mollusk "sticks out". This, by the way, brings it closer to inanimate crystals. On the contrary, the shoot grows with its apical part, which in sunflower is located on the center of the head; all the elements of the pattern (future flowers, seeds) still in the bud of the plant, as it were, are pushed from the inside outwards, moving along the path of least resistance.

## 10. Conclusion

Jacob Bernoulli chose a figure of a Logarithmic spiral and the motto *"EADEM MUTATA RESURGO" ("Changed and yet the same, I rise* *again")* for his gravestone. However, the spiral made by the masons was in fact an Archimedean spiral.

And vice versa, the ever living Spiral Phyllotaxis was mistakenly associated with Logarithmic spiral.


*Fig. 12. Jacob Bernoulli's gravestone.*

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