Drawing as Transformation: From Primary Geometry to Secondary Geometry

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Abstract

A distinction is made between primary geometry, the arrangement in space of lines of projection from a 3-D object to a plane of projection, and secondary geometry, the relationships between the points, lines and shapes of the drawn projection on a 2-D surface. Drawing projection systems, such as those classified under British Standard 1192, are illustrated, and are shown to be defined in terms of primary geometry.

It is argued that a re-classification of projection systems in terms of secondary geometry enables first-year students of drawing to relate more easily such systems of geometry to their observational experiences. Student drawings illustrate the argument.

Drawing Conventions

Following the criteria of David Marr's [1] definition of a representation as a "formal system for making explicit certain entities or types of information, together with a specification of how the system does this", it may be argued that *projective geometry* is such a means of representation, because it provides a formal systematic procedure for making explicit information about the three-dimensional attributes of objects and spaces upon a two-dimensional surface. There are other formal geometric systems which have been devised to represent such information. The various sets of rules which specify how the procedure may operate are termed *drawing conventions.* British Standard 1192 [2] categorises these conventions:

- **Projection**
  - ***Orthographic***
    - Multi plane
      - First Angle
      - Third Angle
    - Axonometric (single-plane)
      - Isometric
      - Dimetric
      - Trimetric
  - ***Oblique***
    - Cavalier
    - Cabinet
    - Planometric
  - ***Perspective***
    - Parallel (1 point)
    - Angular (2 point)
    - Three point

Fig. 1 B.S. 1192 categories of projection types

In this classification, all orthographic and oblique projections may be specified as *parallel projection* systems, since their *projectors,* those lines of projection that link salient features of the object to points on the plane of projection, are parallel. Perspective projections may be classified as *convergent* since their projectors converge on a point in front of the plane of projection, assumed to be a viewer's eye.

Orthographic projection systems

1. *Multi-plane orthographic projection*

   This allows several views of an object to be projected upon several planes, assumed to be at right angles to each other: Projectors are parallel and are perpendicular to the planes of projection. Each object face is parallel with its plane of projection.

2. *Axonometric, or single-plane orthographic projection*

   Projectors are parallel and perpendicular to the plane of projection, and all object faces are inclined to the plane of projection. *Isometric Projection* is a unique case of axonometric in which foreshortening on all three axes is the same. *Dimetric* *projection* is a special case of axonometric in which scales along two axes are equal, the third axis being different. *Trimetric projection* is the general case of axonometric and occurs when all three axes are randomly orientated and are each of different scales.

Oblique projection systems

Oblique projections all have one face of the object parallel to the plane of projection, and the projectors, although parallel to each other, are inclined to the plane of projection in various ways.

1. *Cavalier oblique projection*
   The front face of the object is parallel with the plane of projection, while the projectors from the front face are perpendicular to the plane of projection. The projectors from the other two visible faces, although parallel, are inclined to the plane of projection so that the receding edges are represented at the same true scale as the front face.
2. *Cabinet oblique projection* is similar to Cavalier, except receding edges are drawn to half the scale of the true front face projection.
3. *Planometric oblique projection* is a special case of oblique projection, often inaccurately called 'axonometric', where the plan face of the object is parallel to the plane of projection (and usually rotated through 45º) and projectors are inclined obliquely to the plane of projection.

Two other forms of oblique projection, not identified in the British Standard have been codified by Fred Dubery and John Willatts [3]. They are:

4. *Horizontal oblique projection*. One face of the object remains parallel to the plane of projection and projectors are parallel, but are inclined to the plane of projection *in the horizontal direction only*.
5. *Vertical oblique projection.* One face of the object is parallel to the plane of projection, the projectors are parallel but inclined to the plane of projection in the *vertical direction only*.

Perspective Projection

This family of projection conventions as defined by BS 1192 differs from orthographic and oblique projections because the projected lines from the object to the plane of projection are not parallel, but converge to a point, generally regarded as the position of an observer's eye.

The picture is formed by the intersection of all these projectors with the plane of projection, usually termed the *picture plane* in perspective projections. Parallel edges on the object appear in the projected picture as orthogonals converging to a point, known as a vanishing point.

1. *Parallel perspective*

   The object has its face parallel to and at right angles to the picture plane. Projectors converge to a point.

2. *Angular (2-point) perspective*

   Vertical faces of the object are inclined to picture-plane, horizontal faces remain normal to the picture-plane:

3. *Three-point perspective*

   All the object's faces are inclined to the picture-plane. There are three vanishing points

Primary geometry and secondary geometry

Peter Jeffrey Booker [4] made the distinction between *primary* geometry, the arrangement in space of lines of projection from the three-dimensional object to the plane of projection, and *secondary* geometry, the relationships between the points, lines and shapes of the drawn projection on a two-dimensional surface.

The projection types of B.S. 1192 discussed above are defined in terms of *primary* geometry, but perhaps do not relate easily to students' observational experiences. John Willats [5] has usefully re-classified B.S. 1192 in terms of *secondary* geometry.

For example, in the original B.S. 1192, axonometric drawings showing three faces of an object have to be classified with orthographic projections which show only one face, because their primary geometries have parallel, perpendicular projectors in common. Willats suggests it would be beneficial to re-classify the axonometrics under oblique projections, thus recognising their obvious similarities of secondary geometry, which are the number of faces shown in the drawings, and, the directions of their orthogonals.

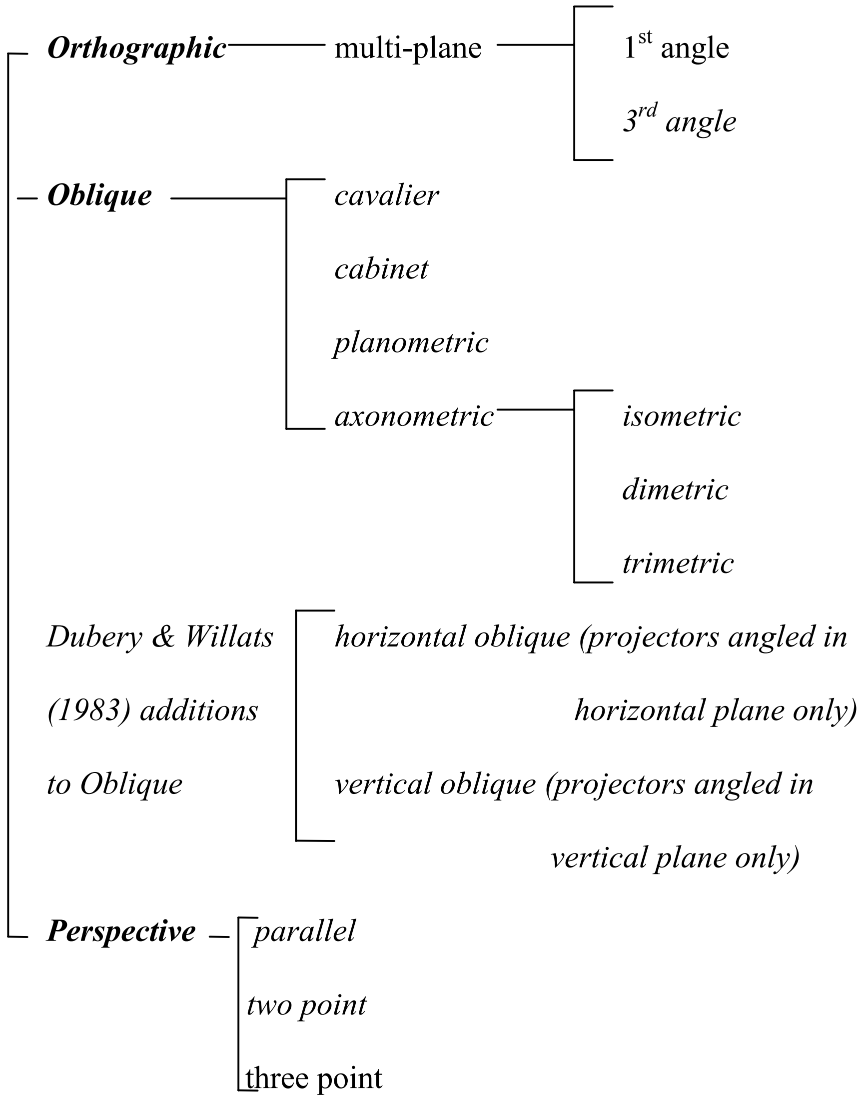


Figure 2 Re-classification of B.S. 1192 in terms of secondary geometry

This re-classification of drawings in terms of their secondary geometry provides a way of understanding those drawings which do not depend upon the drawer's position defined by primary geometry but which, in their secondary geometry, explicate features of the object that are known, but not necessarily visible to the drawer.

Viewer-Centred and Object-Centred Representations

These terms derive from the investigations of Marr and Nishihara [6] into the representation and recognition of the spatial orientation of objects. The two categories are implicit in the classification of projection types. Therefore it may be useful to review those again, this time relating primary and secondary geometries to viewer - and object-centred representations.

According to Marr and Nishihara, vision is the processing of information derived from two-dimensional retinal images (viewer-centred) so as to produce information that allows us to recognise three-dimensional objects (object-centred descriptions).
The organic visual system receives at the retinae constantly changing arrays of light reflected from surfaces and objects in the world from which we derive representations of those surfaces and objects that are consistent, as well as unchanging across varying viewpoints and lighting conditions.
Such representations may take the visible form of drawings not readily classifiable under the rules of primary geometry which are based upon specific assumed viewing positions.
Willats' work over a period of time has synthesised aspects of Marr's theory into a unique approach to the understanding of children's drawings and others whose drawings cannot be defined in terms of primary geometry, but may be understood as examples of the following three categories:

Divergent perspective

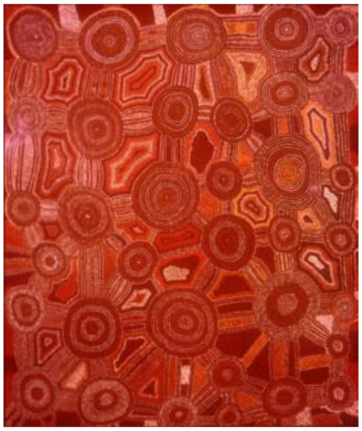
This term describes drawings in which the orthogonals diverge. Although strange to Western eyes, Willats points out that this system, together with horizontal oblique projection, was the most commonly used in Byzantine art and Russian icon painting during a period of over a thousand years. Figure 3 illustrates a more recent example, Picasso's *Woman and*

Mirror, 1937.

Topological geometry

Drawings which map spatial relations such as connections, separation, and enclosure, rather than resemblance and accurate scale, make use of topological geometry. Such drawings may be more easily understood in terms of an object-centred secondary geometry.

Australian aborigine art is often constructed using topological geometry. Figure 4 illustrates the artist Uta Uta Tjingala's painting *Kaakurnatintja* (not dated) which represents the spatial connections between water-holes and other important locations.



"Fold-out" drawings and multiple-view drawings

These drawings display information about various aspects of objects and spaces simultaneously. This is not possible in drawings dependent on single-plane projections based on primary geometry. In Figure 5, Bhawani Das' *Aurangzeb and Courtie's, C1710,* the ground plane has been folded down in orthographic projection in order to convey information otherwise not available from a viewer's position perpendicular to the picture-plane. In the same drawing, the canopy has been rendered in axonometric projection, allowing the viewer a top-view which, whilst inconsistent with the obliquely-projected footstool, affords extra information about the scene.

To continue with the review of projection types in relation to viewer-centred or object-centred representations:

Multi-plane orthographic projection

These drawings are independent of any single viewing position, and are useful for describing the true proportions and relationships between faces of a three-dimensional object. This projection has become the standard for engineers and architects.

Oblique projections

These may be constructed to describe properties of either an object or interior spaces which would not be visible from certain viewer-centred positions. Figure 6 a Punjabi painting *The Gale of Love*, c1810, shows interiors of rooms left and right, which would not be possible in a viewer-centred description.

Types of oblique projection are evident in drawings from various cultures and periods. In the West, an early description of oblique projection was given by Cennino Cennini [7] who advised the artist to

> ...put in the buildings by this uniform system: that the mouldings which you make at the top of the building should slant downward from the edge next to the roof; the moulding in the middle of the building, halfway up the face, must be quite level and even; the moulding at the base of the building underneath must slope upward, in the opposite sense to the upper moulding, which slants downward.

That this advice had already been understood by painters is apparent from Figure 7 painted by Giotto in the *Capella degli Scrovegni* at Padua between 1304 and 1308.

One-point, Artificial Perspective

This is a projection system whose primary geometry is based upon what James J. Gibson [8] termed the *natural perspective* of an array of light reflected from surfaces and converging on the eye. It assumes the viewing position is singular, and static. In terms of secondary geometry, all orthogonals converge on a point known as the vanishing point. Its invention was the culmination of a long-standing desire to produce what Martin Kemp [9] described as "the imitation of measurable space on a flat surface". As such, it may be understood as a more rational codification of the former, loose method practised by Giotto and described by Cennini.

Most authorities agree that linear, one-point perspective was invented by Filippo Brunelleschi in Florence. Kemp [10] cites a source which suggests the date of 1413. It is certain that the system was codified and published in Latin by Leon Battista Alberti in 1435. The Italian version of 1436 had a prologue addressed to Brunelleschi and explained the primary geometry of light rays reflected from surfaces regarded as the base of a pyramid and converging to an apex at the painter's fixed eye.

Students' Drawings

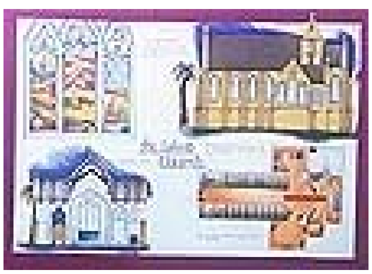
Each one of the ways of drawing discussed above makes certain information about three-dimensional objects and spaces explicit, but at the expense of other information that is obscured.

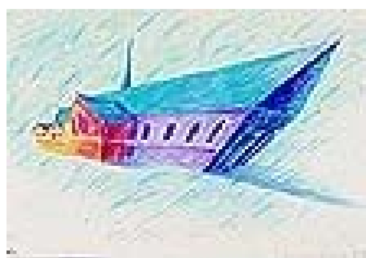
Therefore the choice of a particular way of drawing will depend upon what specific information about the scene, as well as the viewer's position relative to the scene, is deemed important enough to be represented in the drawing. Moreover, such decisions will vary according to the intended purpose of the drawing, for whom it is intended, and according to the socially-conditioned ways that people construe the relationship between themselves and their environment at different ages and in different periods of history.

It is these relationships between drawing and social context that are explored in the drawing studio.

The studio drawing project afforded students the opportunity to relate the concepts of primary geometry and secondary geometry to those of viewer- and object-centred representations through their drawing practice. It may be pertinent to note here that few first-year undergraduates came to the programme with a firm grasp of any geometry , so that for many, this project became an opportunity to explore such basics as orthographic, oblique and perspective projection systems of secondary geometry.

Figures 8, 9 and 10 illustrate examples of such exploration, undertaken as part of a pilot study.





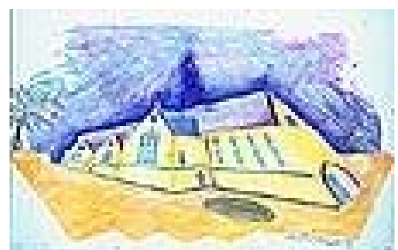


Figure 11 illustrates a *collage* of separate drawings, each a viewer-centred representation of elements within the scene (a set-up of rectangular wooden frames and boxes).

The combination of these viewer-centred representations becomes an object-centred representation, providing information about the scene not available from any single viewing position. It may be noticed that the whole *collage* has been sub-divided along folds which effectively transform the flat plane into a three-dimensional construction, drawing the viewer's attention to the discrepancy between the distal values represented on the drawings' surfaces and the distal values of the three-dimensional scene (i.e. the creased and folded surface). Further evidence of the student's inquiry into geometry provided in Figure 12. This was produced as a result of the student's sustained stimulus beyond the confines of the drawing project itself. It represents a range of systems of geometry, including orthographic projection, oblique projection and vertical oblique projection (the bottle at the right-hand

edge). The combination of high-contrasted tonal shapes in the centre of the painting at the lower end of the dark-toned, centrally-placed vertical axis, produces a variety of depth illusions ranging from shallow to deep. This focal point also offers the viewer an ambiguity of reading; which surface overlaps which?

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