Exploration of Arbitrarily Shaped Surfaces

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Abstract

Exploration of arbitrarily shaped surfaces through linear members using a self-organizing spring system. Similar to the problem with British Court Museum Roof and Anthony Gormley's Body/Space/Frame project, a generative algorithm was developed in order to approximate the geometry of a complex curved surface and optimize the resulting structure.

Regarding the approximation of the curved surfaces, in the thesis, it was aimed to address the problem by using linear members defining an arbitrarily shaped surface as nearly as possible in relation to curvature conditions. In order to tackle this problem, the hypothesis was creating a bottom up self-organizing system that is based on local decisions. By using a bottom-up methodological approach and creating a self-organizing system, it is aimed to overcome the problems that might occur when the curvature is in two or more dimensions. To this end, a generative algorithm that is re-arranging the nodes and providing the connections by using a spring system is developed in Processing. Various experiments are performed in order to determine the closest match of the surface shape and changes of spring length due to curvature. After analyzing the results in the aspect of geometry and topology, results demonstrates that the method is capable of describing an arbitrarily shaped surfaces as nearly as possible in relation to curvature conditions by using linear members.

Keyword: Adaptive, Anthony Gormley ,bottom-up approach, curvature, double curved surfaces, self-organisation, spring system

Introduction

With the emergence of new technologies, cultural and social patterns have started to alter many fields, including architecture. Being aware of those emerging technologies, architects and engineers have started to explore more complex geometries and adaptive forms to reach new design frontiers. In order to solve certain indeterminable complexities and to find an optimum solution during the design process, computational techniques have become more significant.
Considering this recent evolution, this paper focuses on an approach to approximate the geometry of a complex surface through a self-organizing system. A bottom-up methodological approach is chosen, as this research project focuses on the exploration of a curved surface in relation to its curvature conditions and local neighbour relations.
More specifically this research is aiming to develop a generative algorithm that is capable of defining an arbitrarily shaped surface through linear members in relation to the curvature conditions, Considering the complexities that occur when the curvature is in two or higher dimensions, the method proposed here is to create a 'bottom-up' self-organizing system that uses a spring system in order to overcome those complexities.
Even though most approaches are based on top-down methodological approach, the outcome of this projects investigation suggests, that the bottom-up approach is more efficient, especially when it comes to approximating the geometry to its curvature conditions. In the context of this project, approximating the geometry in relation to a given problem, and analyzing it in the geometrical and topological aspects, was crucial.

Theory Background and Related Work

The problem is based whilst building an arbitrarily shaped surface, defining the surface shape as nearly as possible in relation to curvature conditions by using linear members. In this case, linear members' span should vary as having shorter at higher curvature areas and longer at low curvature areas. The question is how to overcome difficulties that might occur when the curvature is in two or higher dimensions.

A similar problem was encountered by Anthony Gormley and Sean Hanna whilst developing the project 'Body/Space/Frame'; the project is a model in the shape of a crouching human figure that consists of 25m high open steel lattices. The aim was creating an algorithm that is defining the position of structural members and their connections in response to the form of the body [1-2]. In order to have optimized structures, the aim was gaining smoothness between curvature and the nodes. Thus, a self-organizing structure was developed in order to approximate the structure and provide structural integrity. In relation to this, he states number of aesthetic criteria that spring system can embody simply. The developed algorithm is including: intermediate number of nodes are distributed randomly in the given boundary of the surface, rearrangement of the nodes due to local forces and the curvature conditions, obtaining an optimal number of connections that is capable of describing the geometry.
The British Museum Court Roof can be referred as similarity of problem in the case of approximating the geometry of an arbitrarily surface. The project was designed by Foster and Partners with the engineers from Buro Happold whereas Chris Williams from University of Bath developed the algorithm. The British Museum Court Roof's

structure covers is 73m and 97 m including the 44m diameter of The Reading Room [4-5]. The algorithm consists of the stages in order to reach the end form of the grid and provide structural accuracy. The first stage of the algorithm is defining the topology of the nodes in order to create a simple mathematical grid. The second stage of the algorithm was created in order to fix the discontinuities, and for this matter relaxation formula is applied iteratively to each point, created in the initial stage. In the sense of optimizing the overall structure, each node was rearranging its position depending on their neighborhood relations. Finally, during the distribution of nodes and rearrangement of nodes by the relaxation process, the curvature of the corners was an important issue in relation to architectural and structural constrains.

It is also worth mentioning the previous MSc. Adaptive Architecture and Computation dissertations 'Topological self-organisation: Using a particle-spring system simulation to generate structural space-filling lattices' prepared by Anastasios Kanellosis, which used a similar approach in relation to the spring force [3]. Briefly, the aim of the project was to fill a certain volume with a structural space frame network lattice. To this end, he developed a self-organising system that consists of certain number of nodes connecting and arranging themselves through local decisions in relation to the given volume. To approach this specific problem, an algorithm using particle system within the framework of physical dynamic simulation was developed. Structure, consisting of particle nodes and spring connections is developed by a simple particle system that has two basic quantities such as a position and velocity vector. Therefore, the spring system was a crucial part of the algorithm as local decisions composed the self-organizing system. ,
Consequently, the proposed method, in order to solve this specific problem, is using a bottom-up self-organizing system that is dependant on a spring system. The algorithm that is developed is using numerical bottom-up method based on the curvature and spring system. By the spring system randomly distributed nodes in the system are rearranged and nodes are connected depending on certain constraints in order to compose the linear members. Furthermore, whilst arranging the given number of interconnected nodes in relation to certain constraints, topological and geometrical aspects are crucial. Therefore in the stated problem, investigating the topology of the geometry in order to generate a structure with the approximate number of nodes is one of the key considerations.

Method

Overview of the algorithm

Referring to the problem, which is composing an arbitrarily shaped surface by distribution of linear members in relation to curvature conditions that is describing this
specific surface as nearly as possible, the hypothesis is solving the problem by creating a self-organizing system that is based on local decisions. The aim is to create a self-organizing system that is capable of generating the geometry and topology of an arbitrarily shaped surface. To achieve that, an algorithm was developed in certain concerns: intermediate number of nodes are distributed randomly within the given boundary of the surface. The nodes are then rearranged due to local forces and the curvature conditions, obtaining an optimal number of connections, that is capable of describing the geometry, are obtained.

The reason that the distribution of the nodes should be according to curvature is in

order to obtain a precise mesh generation in relation to the geometry of the surface. Neighbour relations can be described as the interaction between nodes and the connectivity pattern in the system that is based on the spring force. Spring force optimises the nodes' position and its relations whilst forming the temporary bonds between the nodes. The permanent bonds between the nodes compose the final form once it is in a certain equilibrium state. In relation to bottom-up approach, connections of the nodes and the position of the nodes are not determined in the initial phase rather they form dynamically during the process as it is a self-organizing system.
The base of the algorithm as the equation of the Bezier surface is adapted from Alasdair Turner's Bezier surfaces code. The development of the algorithm for the stated problem was achieved by the Processing programming language.

Description of the algorithm
Following the methodological consideration in relation to the problem, the algorithm features two main parts: representation of the Bezier surface and creation of the mesh structure. Therefore, the algorithm is divided programmatically into three parts: the programmatic definition of a Bezier surface, calculation of the curvature and setting the nodes according to spring force. Furthermore, there are two main classes that are responsible for creating surface points which are based on the Bezier equation and given u, v coordinates. The u, v coordinates are represented within a two dimensional local parametric space that is situated in three-dimensional Cartesian geometric space. Curvature and spring force are also set in the class of Bezier surface within the local parametric space, as node distribution and connectivity pattern is dependant on those specific local decisions.

In initial phase, the Bezier surface was set as an array of points, ***pt***, according to the *u*, *v* coordinates within a two dimensional local parametric space. The points that are set within the *u*, *v* coordinates were basicaly based on the parameters of the control points, ***ctrl_pts***, that are set in the Cartesian coordinate system [6]. At first points were distributed evenly on the Bezier surface, composing a grid. However as bottom up approach does not require explicitly defined points, second attempt was distributing the points randomly within the local parametric space (Figure 01).

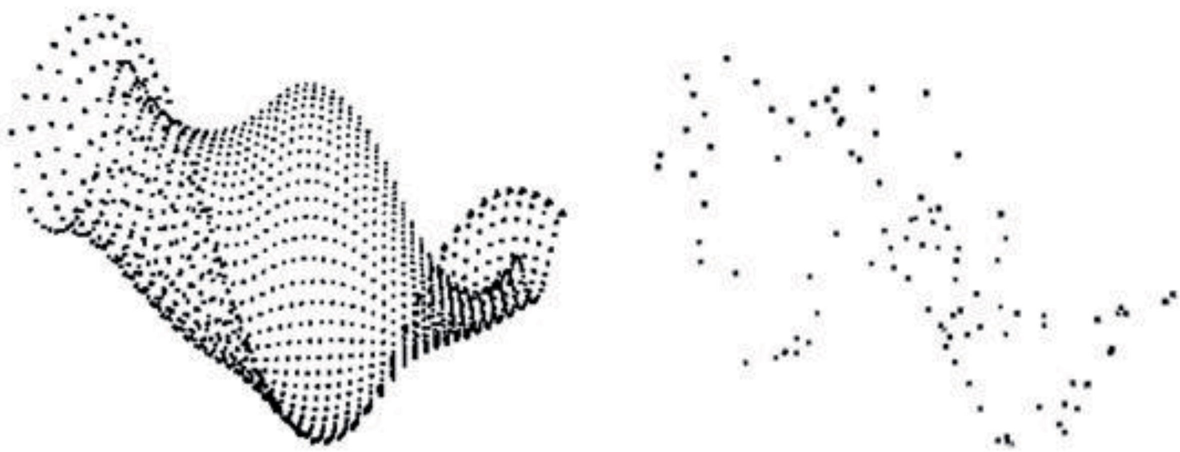

Figure 01.
Bezier Surface; nodes on grid and nodes distributed randomly on the u,v coordinate

system

After distributing the point randomly, local curvature conditions of the Bezier surface were calculated. The final step of the algorithm is the composition of the spring and calculation of the spring force in order to rearrange the position of the nodes and create connectivity patterns, accordingly.

Briefly, the approach of the spring force similar to Kanellos was developing an attraction and repulsion force between nodes in relation to their proximity demand. Following to the proximity check, the next step was creating the connectivity pattern between nodes. The calculation of the spring system and the connectivity pattern varies due to its dependence to the distance between two points, d1 or distance between the surface point and the average of two points, d. Within randomly located points, two points, ***pt1*** and ***pt2***, were picked. The distance, ***d1,*** between those two points, ***pt1*** and ***pt2***, was calculated, as it will be used in the calculations at stage of nodes interaction. There was another essential distance that should be calculated in relation to configuration of spring force. Therefore, specific point, ***pt3,*** from the surface points according to u, v coordinates was picked. After calculating the average of pt1 and pt2 and setting as ***pt4,*** the distance, ***d***, between pt4 and pt3 is calculated. Those distances, d and d1, are crucial in relation to the spring as the geometrical and topological analysis will be dependant on them in order to evaluate the formation of the structure. In order to define the spring force equation, a certain spring threshold and a connection distance are defined. Following that, the equation for repulsion and attraction of nodes were set in relation to spring threshold (Figure 02). During the repulsion and attraction of the nodes, in order to show springs defined with temporary connections. The permanent connections between the nodes occur when the distance, d1, between two points, pt1 and pt2 is less than the connection distance (d1 < connectDist) as the system starts to settle.

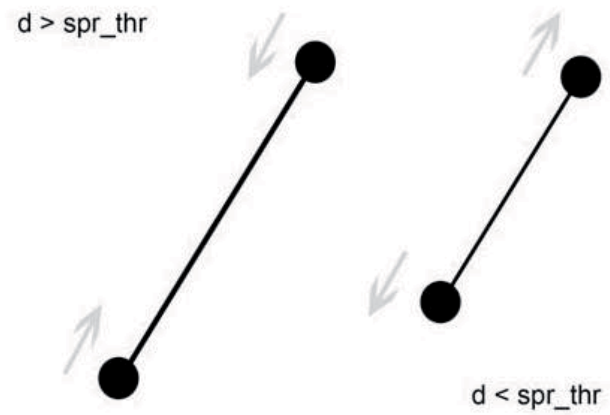

Figure 02. Two cases of spring force

Analysing the equation of spring force, the calculations vary as the spring is calculated either in relation to strut, the distance, d1, between pt1 and pt2 or the distance, d, between surface, pt3, and p4.

Testing and Results

During the experiments, alterations in the system were carried out in order to determine the best match of the surfaces shape in relation to local decisions and curvature conditions. By changing the parameters of the connection distance and spring threshold, the system is challenged to settle with the pre-eminent surface shape match.

The performance of the algorithm, in order to quantify the closest match of the surface shape was based on the calculation of the average derivation of the spring length from the mean length. As explained in the methodology section, the spring is calculated either in relation to strut, the distance, d1, between pt1 and pt2 or the distance, d, between surface, pt3, and p4. Thus, during the experiments the algorithm was run separately with the distance, d and d1 in order to calculate the mean spring deviation accordingly. Also, during the experiments the change in the distance, d and d1, spring derivation was observed in relation to connection distance and spring threshold. By changing the parameters of the connection distance and spring threshold, system is challenged to settle with the pre-eminent surface shape match.

First experiments were performed with the same curvature in order to observe the mean spring length deviation in an equally challenged environment both in u,v coordinate system and Cartesian coordinate system (Figure 03).

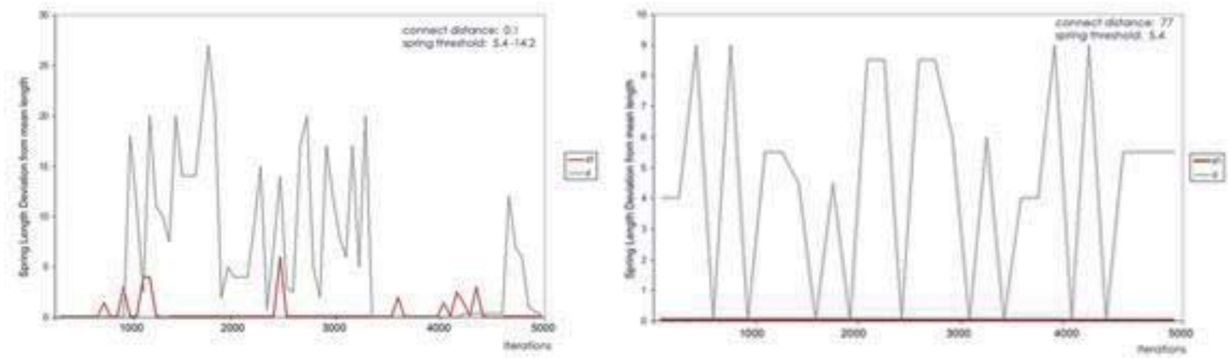

Figure 03.
Left: The Average Spring Deviation from mean length in respect to d and d1 during run time of the algorithm (connect distance: 0.1, spring threshold: 5.4-14.2) based on u,v coordinate system
Right: The Average Spring Deviation from mean length in respect to d and d1 during run time of the algorithm (connect distance: 77, spring threshold: 5.4) based on and Cartesian coordinate system

Two different variables that were used to calculate the spring were run separately in same conditions and represented in the same graph in order to show their difference and spring deviation in each case. During the both runs parameters of the connection distance and spring threshold were also altered in order to observe system's response and achieve the best result with certain parameters (Figure 04, 05).

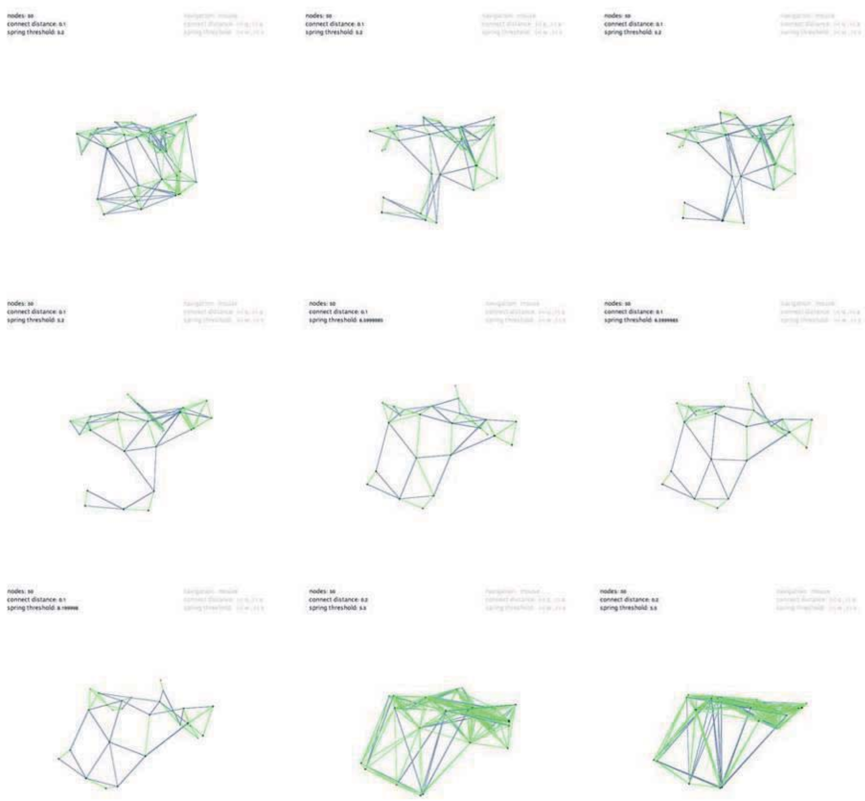

Figure04. Illustration of the surface during the algorithm run (average spring deviation in respect to the parameter 'd1' based on u, v coordinate system)

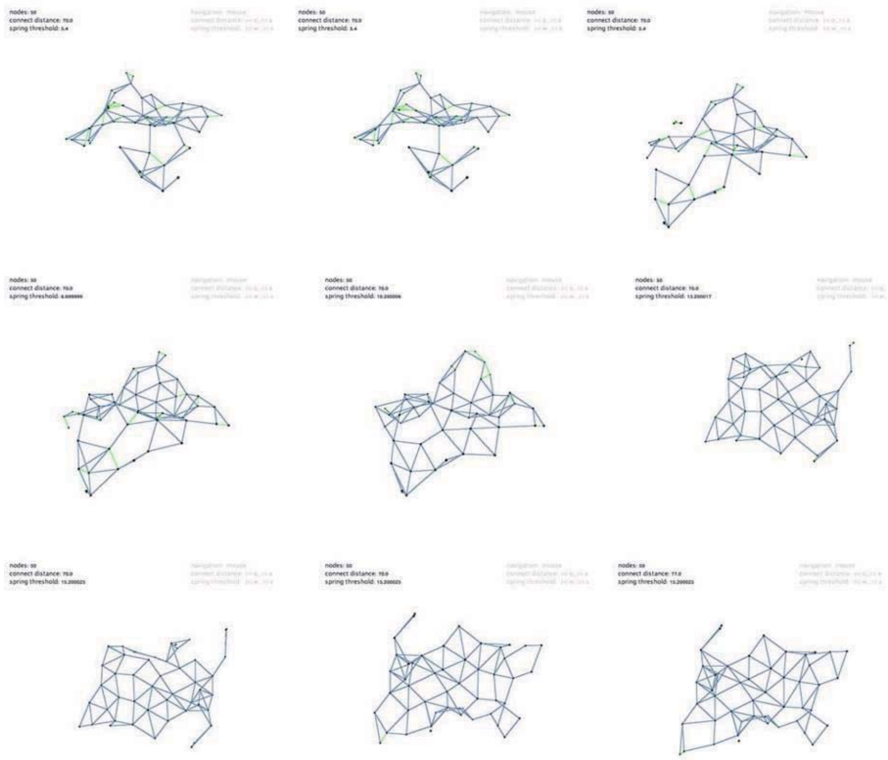

Figure 05. Illustration of the surface during the algorithm run (average spring deviation in respect to the parameter 'd' based on Cartesian coordinate system)

After the first experiments, evaluation of the system is performed in relation to the distance d and d1 in order to quantify closest match to surface boundaries. Subsequently, second experiments were performed according to the best results that were obtained from the previous tests and run again with same curvature and with a higher curvature in order to observe the spring deviation change and evaluate system's response to higher curvatures (Figure 06, 07).

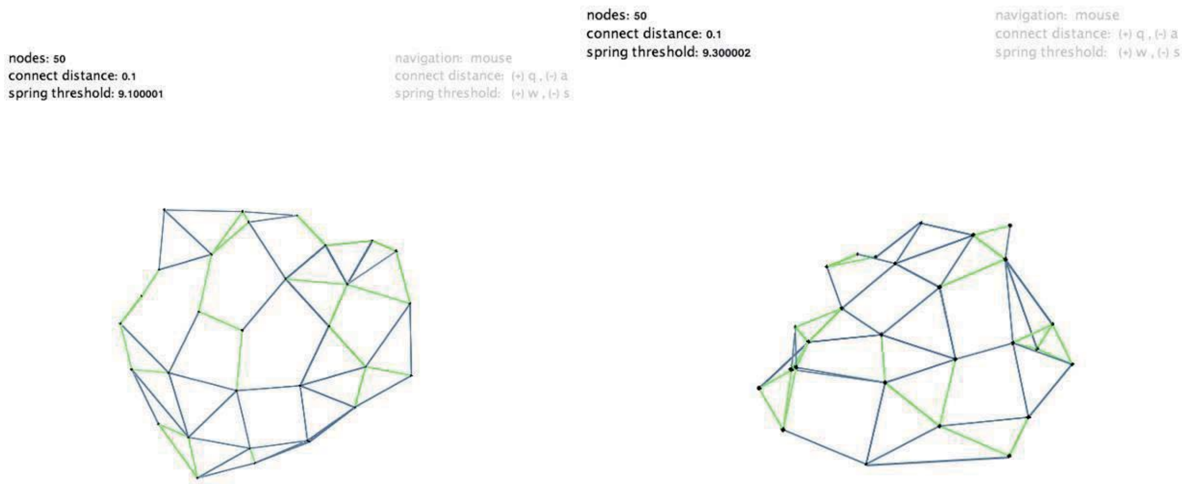

Figure 06.
Left: Illustration of the surface during the algorithm run presented in figure 22 (average spring deviation in respect to the parameter 'd1' based on u, v coordinate system)
Right: Illustration of the surface during the algorithm run presented in figure 23(average spring deviation in respect to the parameter 'd1' based on u, v coordinate system) higher curvature conditions

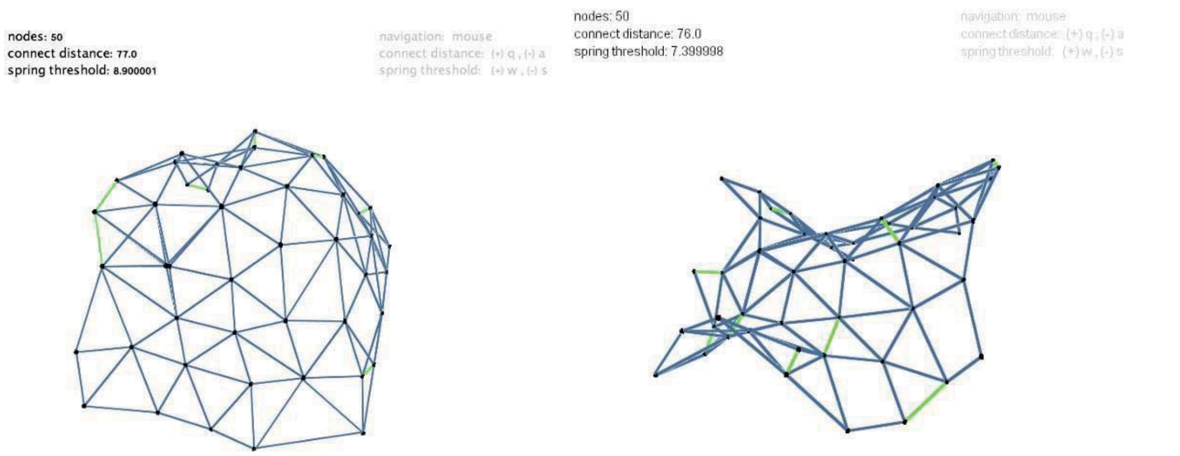

Figure 07.
Left: Illustration of the surface during the algorithm run presented in figure 24 (average spring deviation in respect to the parameter 'd' based on Cartesian coordinate system)
Right: Illustration of the surface during the algorithm run presented in figure 24

(average spring deviation in respect to the parameter 'd' based on Cartesian coordinate system) higher curvature conditions.

Evaluation

The problem that was set at the beginning was building an arbitrarily shaped surface by using linear members within the concern of curvature conditions and defining the surface with those linear members as nearly as possible. Accordingly, linear members' span should vary in higher and lower curvature areas. Regarding the problem that was defined hypothesis was creating a self-organizing system in order to overcome the problems that might occur when the curvature is in two or more dimensions. Consequently, referring to the results, the algorithm, which is based on a bottom up approach, is capable of creating a frame structure within the surface boundaries as nearly as possible in relation to curvature conditions.

The descriptions that are related to the curvature as providing longer members at lower curvature areas, shorter members at higher curvature areas and increasing the node density throughout the surface, are within the top-down approach studies. In this case, by using a bottom up self-organizing system the generative principles behind those descriptions were represented by providing a simpler explanation. Basically, nodes' density throughout the surface and distribution of the nodes in relation to the curvature were accomplished by using a spring system. The deviation was checked both in u and v direction, and by this way the exploration of the surface was more profound in the sense of providing a closer match of the surface shape.

Throughout the experiments, average spring length deviation was observed in relation to curvature changes, and after determining the parameters that forms the best match the surface shape, the tests were run one more time. Referring to final experiments that are performed in various curvature conditions, it can be stated that the algorithm's challenge was rearranging the nodes at higher curvature areas where spring length deviation was increasing. However, after a while the system was settling down and providing a distinguishable pattern. Finally, it has been observed that the system is forming triangulations rather than quadrates, especially in higher curvature areas and defining the geometry more accurately.

Conclusion

In regards to the approximation of curved surfaces, the thesis aimed to address the problem by using linear members defining an arbitrarily shaped surface as nearly as possible in relation to curvature conditions. In order to tackle this problem, the hypothesis was creating a bottom up self-organizing system that is based on local decisions. In order to examine the relation between spring length and curvature, and quantify closest surface match to surface shape, certain experiments were performed. During the experiments, certain assumptions were formulated which led to a more precise definition of the system's parameters and algorithm steps. The tests were run again based on the parameters that provided better results. Several quantitative features of the created the structure, consisting of linear members defining the surface shape, were documented as plots and visuals.

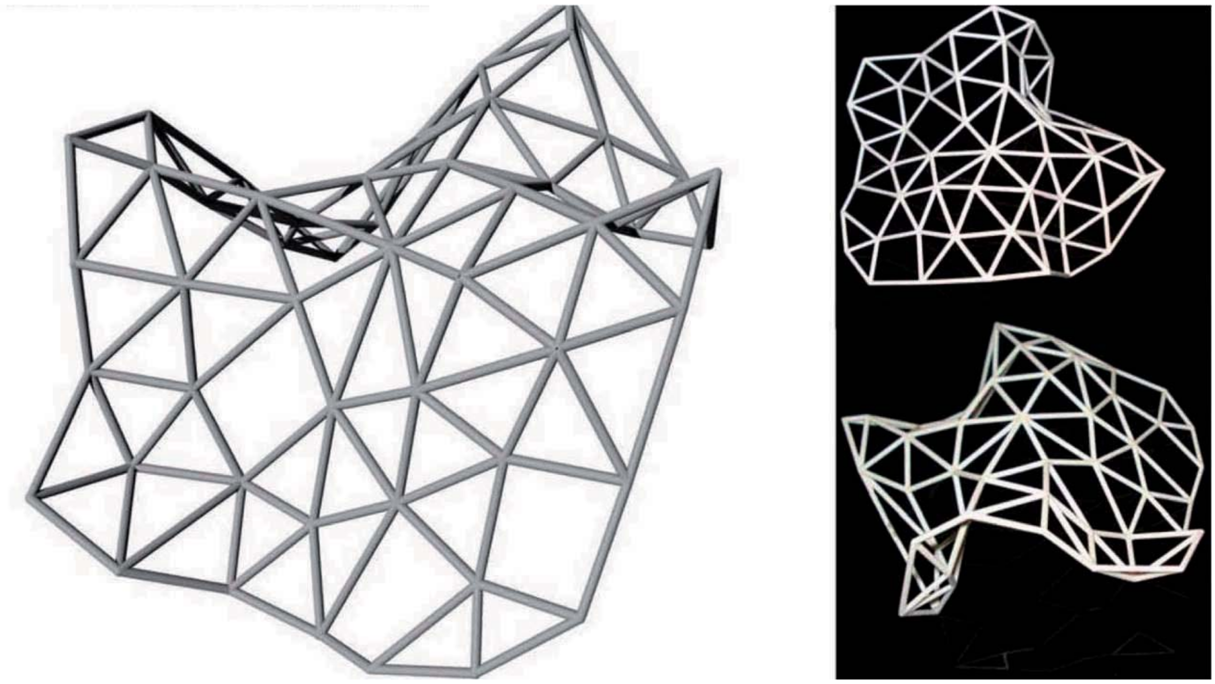

Figure 07.
Left: *The end result, the geometry obtained in Processing is exported to Rhino.*
Right: *The end product, obtained through 3D fabrication method.*

And finally, in order to investigate the results more precisely, the data of the finalized structure was imported from Processing into Rhino (Figure 07, Left). This led to the development of a rapid prototype model, to help to examine its structural performance (Figure 07, Right). The evaluation of the experiments and results indicate that the proposed algorithm was successful, in the sense that the linear members were defining the surface shape as nearly as possible.

As possible future development, the generated topologies could be focused on structural aspects. By applying a force to the system, it will be capable of adapting itself to more complex conditions in relation to the deformation behaviour of it's members. In this case the application of the force should come from the top, onto the areas with a higher curvature in order to define the weak points in the system and rearrange the node distribution accordingly. This implies a revaluation of the spring system has to be done. Through the force application, the system will be able to confront more complex geometries and by this it will also become capable of adapting itself to a certain amount of deformation. As a result of those improvements the algorithm will become more valuable as it will not only consist of geometrical and topological, but also of structural aspects.

As future context, generated topologies can be focused on structural aspects. By applying a force to the system, system will be capable of adapting itself to more complex conditions in relation to deformation behaviour of members. In this case the application of the force should be on the top, to the higher curvature areas in order to define the weak points in the system and rearrange the node distribution accordingly that means also revaluation of the spring system. By force application system will be able to confront more complex geometries and by this it will be capable of adapting itself to certain amount deformation. Finally, by those improvements the algorithm will be more valuable as it will consist geometrical, topological and structural aspects.

Credits

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