A Shape Grammar Model To Generate Islamic Geometric Pattern

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Abstract

Geometric patterns are the important ornaments of Islamic art and architecture. Polygons and poly-pointed star patterns, especially the 6-, 8-, 10- and 12-sided polygons were mostly used in Islamic art and architecture. They have created various geometrical compositions on the surfaces. The 10 sided polygon, decagon, is a special shape in Islamic geometry. This study is evaluating the special features of decagons in Islamic art by analysing the Penrose tiles, aiming the generation of new patterns that have the similar characteristics of the geometric patterns in Islam with the help of a shape grammar model. In this context, two design templates developed are for the new generated patterns.

In this paper, the combinations of the bowtie and the elongated hexagon, which are the sub shapes of a decagon, are mentioned and generated to make it possible to cover a rectangular surface. Two bowtie-elongated hexagon templates help to generate new forms. They are related to Islamic geometrical patterns and many results of the generation from these two templates are given in the framework of this study.

The aim of this paper is to show that different final products can be formed from the shapes, which have the same design language and same initial shape applying the same rule schema. It is clarified that a design can be handled at a wider perspective in the context of shape grammars and reproduced by the new designs with the same genetic features.

The future goal of this research is to apply this shape grammar model and the design templates with a computer program, in order to get the results easier and faster.

1. Introduction

Since 1977, Stiny’s paper about generation of Chinese lattice design [10] shape grammars have been used to understand the design language, to analyse a design or a shape and to generate new designs from the same language. The shape grammars are a rule-based design method, which helps to solve the rule schema of a structure of a design. A lot of houses, shapes or compositions such as Turkish Traditional Houses [6], Alvaro Siza’s Malagueira Houses [7], Queen Ann Houses [8], Palladian Villas [11], Mughul Gardens [12], Architect Sinan’s Mosques [13] and the Meander Motifs on Greek Geometric Pottery [9] were analysed in the context of
shape grammars. The analyses of Islamic geometries were also a research field in different disciplinary [14-17].

Before the acceptance of the religion of Islam there was not a significant art style or a dominant architectural tradition. After that, there was the belief coming from the respect to the principle “There is no God but Allah!” which prohibited using the figures as decoration elements. In Islamic art and architecture, the role played by the non-figural style in religion caused the patterns to come in three distinct geometrical types: the Arabic script, floral decoration called Arabesque, and geometrical patterns [4]. First recognizable style is the Kufic patterns that are formed into rectangles and squares to create calligraphic designs. Such patterns are used to add dignity and solemnity onto the architectural surfaces. The second distinct pattern group is the floral decoration, which is also called as arabesques. This type of ornaments occur curvilinear elements symbolizing leafed and floral forms [1]. The significant ornament type in Islamic art and architecture is the geometric patterns that include complex polygons and poly-pointed star shapes. The patterns were applied on different types of materials such as brick, wood, tiling, stone, plaster, stucco and paper. Islamic compositions are usually used for wall, door, window, ceiling and mimbar decorations.

There are numerous different geometric patterns in Islamic art and architecture. This study focused on the geometric compositions, their sub shapes and the generation of the new geometric patterns, which have the same characteristics with the original ones, based on a shape grammar model.

2. Geometric Patterns

The most striking characteristic of the geometric patterns in Islamic art is the prominent symmetric shapes, which are like stars and constellations. 6-, 8-, 10-, 12-pointed star shapes are the ones that occur mostly in ornaments. Another important characteristic is that the rectilinear forms are interlacing each other, and continuously following the lines. The last features are the variety of the shapes, the flow and unboundedness of the entire composition. In the composition, the context of copying the shape is that, every cell repeats itself indefinitely with the rule of symmetry to fill the space [1].

The most popular shapes such as 6-, 8-, 10-, 12-pointed stars are created in the limits of the hidden circle. The circle helps to perceive the interlaced composition as a strong unitary, which symbolizes the one and the only God, Allah [3]. The geometries develop in two different ways such as linear-developed compositions and centric-developed compositions. The first type of composition includes zigzags, V-formed shapes, meanders and also poly-pointed stars. The centric-developed compositions, also called closed geometries, contains polygons, poly-pointed stars, medallions and mixed geometries.

Below it is seen the creation of a hexagon and a 6-pointed star; an octagon and an 8-pointed star; a decagon and a 10-pointed star; and finally a dodecagon and a 12-pointed star in 3 different ways. First 3 shapes are created by centric rotation of the same circle after starting by a point, a line and then a simple shaped circle. The
figure 4 shows that the initial shape is different for the same rule schema that creates the same final shape in different scales.

Figure 41. Creation of hexagon and 6-pointed star.

Figure 42. Creation of octagon and 8-pointed star.

Figure 43. Creation of decagon and 10-pointed star.
3. Pentaplex Tiling and the Special Shape: Decagon

It is a common method to use various geometric tiles to cover a surface. The assembling of the tiles establishes an important point which is, the tiles cannot overlap on each other, and there cannot be a gap between them. The figure 5 shows the most used tiles in Islamic ornaments and makes clear that equilateral triangle, square and hexagon have a rotational symmetry, and they can cover a surface
without any gaps or overlapped tiles. However, the pentagon and decagon need support elements to close the rotational symmetry. Therefore, it is not possible to create a periodic pattern with rotational symmetry using only pentagons or decagons.

There are seventeen periodic pattern types created by 2-, 3-, 4- and 6-fold rotational symmetries. Most of them are also used in Islamic geometric patterns [1]. But the 5-fold rotational symmetry is not used for the periodic patterns. From this point of view, the aperiodic patterns with 5-fold rotational symmetry could be discussed.

The Pentaplex tiles are mentioned first in 1970’s by Roger Penrose, a famous physician. These tiles cover the surfaces, which contain 5-fold rotational symmetry, aperiodically. The Pentaplex tiles in Turkish-Islamic art have been being used since 11th century [2]. The figure below shows one of the 3 types of Penrose tiles. This group contains the shapes called the kite and the dart.

![Figure 46. The alternative Penrose tile combinations of darts and kites.](image)

When the entire composition created by Penrose tiles is researched, and the assembling of the sub shapes is observed, it is realized that kites and darts create new shapes, which will be the key points of the design templates at the end of this research. The figure 7 clarifies the very first steps, and the source of the entire study.

![Figure 7. The creation of the decagon from the tie and the bowtie.](image)

1st step shows the sub shapes of the Penrose tiles, the combination of darts and kites. At the 2nd level three new shapes called tie, fish and star are created. The next level makes it clear that the tie is also the sub shape of the fish and star with a new
shape named bowtie, which is shown with the color light grey at the 4th step. The last step is the most important one that represents the creation of the special form decagon from the tie and the bowtie. In other words, the decagon comes from the sub shapes of the Penrose tiles, and also it can be said that the sub shapes of the Penrose tiles are also the sub shapes of the decagon.

4. Pattern Generation from Decagon

The figure below shows that the tie and the bowtie come together in many variations to create fish, decagon and star shapes. There are 2 variations for the fish shape, 10 variations for the decagon and 5 combinations for the star shape.

![Figure 8. The variations of tie and bowtie](image)

![Figure 9. 5 girih tiles: decagon, pentagon, hexagon (tie), bowtie, and rhombus [5].](image)

In 2007, Peter Lu and Paul Steinhardt mentioned in their paper about the Decagonal Tilings in Medieval Islamic Architecture that equilateral polygons –girih tiles- created complex periodic girih patterns with self-similar transformations to construct nearly perfect quasi-crystalline Penrose patterns five centuries before their discovery in the West [5]. The drawing on the left is the complete set of the 5 girih tiles: decagon, pentagon, hexagon (in this paper it is called as tie), bowtie, and rhombus. The picture on the right hand side in Figure 9 is from the outside wall of a 15th century Timurid tomb with tie, bowtie and decagon layer on it. The overlapped layer indicates that this pattern is developing periodically. From this point on, it can be said that it is possible to create periodic patterns by using only decagons, which consist of three ties and a bowtie.
First drawing below illustrates the generation of the 1\textsuperscript{st} design template. First step shows the initial shape of the shape grammar. The decagon, the initial shape, is rotated on the corner 5 times intersecting each other in the context of the rule 1. The second shape is a combination of ties and fish shapes with 5-fold rotational symmetry. According to the rule 2, two decagons are added above the last generated shape at the edges. Rule 3 contains the reflection of the entire shape vertically before it is reflected horizontally in the rule 4. After the rule 4 was applied there is a star-decagonal formed gap in the center. Last rule tells to put a decagon in the center of the whole composition. The output shape is the first design template that will be covered by covering tile groups afterward.

![Rule Schema of the 1\textsuperscript{st} Design Template's Shape Grammar](image)

\textit{Figure 10. The rule schema of the 1\textsuperscript{st} design template’s shape grammar.}

Second drawing shows the generation of the 2\textsuperscript{nd} design template. The rule schema is similar to the first shape grammar’s rules. The main point of two rule schemata is the symmetry that is also the most important feature of Islamic patterns. Another key point is to put the decagons at the corners and in the center of the entire design template. The last thing to pay attention is the symmetry axes of the whole pattern have to cross the symmetry axes of the shapes on it. Figure 12 indicates that the edges of the rectangle and the axes of the pattern are located on the symmetry axes of the star shape, tie and bowtie.
The design templates can also generate themselves in the context of the rules and constraints. As it was mentioned before, star, fish shapes and decagons consist of ties and bowties, so that for every decagon there are 10; for every star shape there are 5; and for every fish shape there are 2 different combinations. There are also combinations of every shape with each other that give numerous alternatives from two design templates. The Figures 13 and 14 show some alternative results of the two initial design templates. The code of the derivatives indicates the number of the design template • (number of the level•) [number of the derivation at the 1st level • number of the derivation at the 2nd level • number of the derivation at the 3rd level] • the letter code of the derivative at the 4th level.
Figure 13. The derivatives of the 1st design template.

Figure 14 The derivatives of the 1st design template.
There are three groups of covering tiles to fill the end derivatives. Figure 15 shows the tile groups for each decagon, tie and bowtie. First group is developed from Lu and Steinhardt’s researches. After they examined some ornaments of Islamic architecture such as Darb-I Imam Shrine at Isfahan, Iran, Ottoman Green Mosque in Bursa, Turkey, Mughal I’timad al-Daula Mausoleum in Agra, India they found many geometric patterns combined by decagon, tie and bowtie. The first group is the simplest one of all. 2nd and 3rd groups are created by Prof. Dr. Metin Arık and Mustafa Sancak. The 3rd one is the most complex one and involves more tiles than the other group.

First column of the groups shows the covering tiles for the decagon shape, by 2nd column for the tie and the 3rd column for the bowtie. That means where there is a decagon in the design template it will be filled by one of the circular patterns, if it is a tie then it will be covered by one of the vertical patterns, and a bowtie will be filled with one of the horizontal patterns.

It is given in the figures 16 and 17 an end derivative from the 1st design template and the geometric pattern results after all of the decagons, ties and bowties filled with every group of covering tiles. Figure 18 and 19 show the same results for one of the end derivatives of the 2nd design template.
Figure 16. Derivative of the 1st design template covered by 3 groups of covering tiles.

Figure 17. Derivative of the 1st design template covered by 3 groups of covering tiles.
Figure 18. Derivative of the 2nd design template covered by 3 groups of covering tiles.

Figure 19. Derivative of the 2nd design template covered by 3 groups of covering tiles.
5. Conclusion and Future Studies

In the context of this research the special features and the creation of decagons are evaluated by analysing the Penrose tiles. With the help of a shape grammar model the structure rules schemata is developed to generate new patterns that have the similar features of the geometric patterns in Islamic art. From this point, it is given two design templates, which are developed for the new generated patterns in the context of the characteristics of Islamic geometries.

In this study, the shape grammar model helps to determine that it is possible to get different final products, which have the same initial shape and design templates created with the same rule schemata. This result indicates a design analyses could be dealt with in a wider perspective and generate new designs with same characteristics.

According to the analyses it is realized that the decagons are acting different than the other shapes creating geometric patterns in Islamic art and architecture. The decagons with a 5-fold rotational symmetry make it possible for the sub shapes, tie and bowtie, to create countless template variations for Islamic patterns. When the structure of the decagons is examined, it clarifies that ties and bowties have been being used for hundreds of years in Islamic art and architecture. It must be accepted that all of the geometric patterns in Islamic art and architecture are perfect results of a superior intelligence. This study shows that these patterns do not have limits to be designed, and they can be generated in the context of lots of different rule schemata. They never could be regarded as old fashioned or unusable.

For the future studies the shape grammar model and the design templates will be applied into the computer to get the results easier and faster. The model could be changed by the new rule schemata, and the new design templates and the new covering tile groups could be generated with help of the computer software. Another goal for the future is to get digital results intended for manipulating and manufacturing the patterns in the sense of the computer generated pattern program, which could be used not only for the Islamic patterns but also for many existent or new geometric patterns.

References


