Impossible figures of Yturralde: A challenge in 3D

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Abstract

Jose Maria Yturralde, Spanish National Prize of Fine Arts in 2020, was one of the pioneers generating art with computers, as early as late 60's of last century. Some of his famous paintings of impossible and computable forms were produced in the late 60's and early 70's.

Our challenge is to make possible in 3D some of his 2D impossible figures. In order to do so, we transform the apparent planar surfaces into hypars (hyperbolic paraboloids) so that, from the suitable point of view, they look exactly like the original 2D paintings, but once they are rotated it is possible to appreciate the 3D realisation of the impossible figures.

Figura imposible (Yturralde, 1972)

The software that we use in order to generate the 3D forms is Wolfram Mathematica. This software allows us to parametrize the different hypars that compose the forms, produce a virtual representation of them that can be rotated dynamically, or they can be exported to several formats that can be used in CAD software, or be sent to a 3D printer.

The hypars are doubly ruled surfaces, that is, for any point of the surface two different lines pass that are contained in the surface. With Wolfram Mathematica we use a macro for ruled surfaces which is easily adapted by only changing the four ordered points that define each hypar. The main challenge for each form is locate adequately each quadruple of points associated to each hypar face of the figure.

Figura imposible (Yturralde, 1971)

In the format of virtual Poster, we show all the details and mathematical formulas to produce our 3D realisations of the selected paintings of Yturralde. The way they are generated allow the reproduction of the forms with other software which includes the possibility of parametric generation.

This work was produced under the advice of Professor Alfred Peris, from the Universitat Politècnica de València (Spain). We want to thank Professor Yturralde too, for all the interesting discussions and help that we received during the selection and 3D generation of his impossible figures.

To generate this impossible 3D figure, we have defined the following command in Mathematica:

```
RuledSurface[alpha_, beta_] :=

  ParametricPlot3D[{v*alpha[u][[1]] + (1 -
v)*beta[u][[1]],

    v*alpha[u][[2]] + (1 - v)*beta[u][[2]],
```

v*alpha[u][[3]] + (1 - v)*beta[u][[3]]}, {u, 0, 1}, {v, 0, 1},

Mesh -> None, PlotStyle -> Blue]

Subsequently, we have selected key points to define the lines alpha and beta, and with them, we have defined the corresponding surfaces. This was done with the surfaces that are above the diagonal formed by the lower-left vertex and the upper-right vertex.
Finally, we have defined the rotation matrix:
A := {{Cos[Pi], Sin[Pi], 0}, {-Sin[Pi], Cos[Pi], 0}, {0, 0, 1}}
To find the symmetric part and combine them to achieve the 3D figure.