How to teach Concrete art to a robot?

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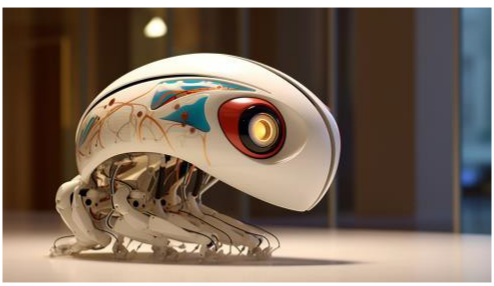

Boustrophedon (tutor: Laurent Daniel)

Summary

This article explores the challenges related to training a robot designed to produce generative Concrete art and which ultimately must not be dependent on learning based on phenomenal amounts of data. This approach is also ideal for studying the formalization of the artistic creation process using a machine: the robot.

The article will first clarify the notions of chaotic and random processes which are closely linked to the nature of algorithms entering the robot's knowledge base. Subsequently, the article introduces the differentiation between Synthetic reasoning programming allowing emulated reasoning via a process of Gödelization, and Graphic programming as an anthropomorphic vision of a robot knowing how to draw and color and finally, the generation of generative art by artificial intelligence.

1. Concrete Art

“Concrete art proposes to replace artistic imagination with mathematical design". If this postulate of Max Bill, which mainly applies to the mathematical principles of shapes and colors, has since become the manifesto of the Concrete art movement, we find older graphic and pictorial examples which perfectly respect the postulate. We refer, for example, to the system of codification of Aztec shields (figure 1) allowing visual identification of the rank and identity of the warrior.

2) Self-coded and self-encrypted Art

The dual cryptographic and aesthetic approach which led to the graphic and pictorial design of the shield invites us to examine concrete art from the angle of coded information without encryption keys or encrypted information with keys.

Figure 1. Aztec Shield (Source: Wikipedia Commons CC by ShareAlike 4.0 International)

In this respect, the systems elaborated by Julio Le Parc or Victor Vasarely to encode colors are true systems of cryptography based on combinatorics and colorimetry. Vera Molnar's work constantly oscillates between the visual result and the artist's algorithmic creative process which is often hidden but sometimes revealed for in a pedagogical way by the artist. In view of the major problems related to the legal protection of IP rights of artists, it is useful to think of the algorithmic choices of a robot, as well as the parameters associated as encryption keys. We are thus moving towards the concept of self-encoded and self-encrypted art.

3) Robots that think and draw

Programming languages, well as algorithms and information theory are at the heart of the interdisciplinary dialogues between artists and mathematicians.

Figure 2. Maurice El-Milick, Albert Ducrocq. (Source: private collection)

"The operations of concrete art (repetition, progression, permutation and combinatorics of all kinds) are in fact emblematic of the new computational thought that accompanies developments in cybernetics and information theory" [5] (translation).

A wonderful historical retrospective of Art Tropism towards programming is presented in [1]. We shall not forget that this movement brought together artists towards artistic crafts by means of mechanical programming intended for weaving looms (Anni Albers, Gee's Bend movement) for example. Some articles concerning Vera Molnar [4] allow us to better understand the connection between art and mathematics. However, the reciprocal movement of mathematicians or roboticists towards art through programming languages is less known and documented. From 1936, and therefore before the appearance of the very first computers, Maurice El-Milick formalizes a graphical, symbolic, and purely mathematical programming language that he calls ornamental algebra. This fully functional language in modern programming environments uses the formalism of so-called explicit equations (figure 2 left) and the theory of transformation groups to produce abstract or figurative ornamental art. In 1953, the roboticist Albert Ducrocq (figure 2 to right) uses binary language to develop the Calliope robot, a text and image generator [2], thus inventing “prompt engineering” several decades before its rediscovery by the generative art systems powered by artificial intelligence. Subsequently other programming languages helped to establish and reinforce the links between mathematics, graphic design and art. We will briefly cite the Lisp language at the heart of the epic “Thinking Machines” of the early 1980s, then the Postscript language used at the same time by the mathematician Henry Crapo to create mathematical figures of projective configurations, paving the way and laying the foundation of "Geometric Reasoning" [1], subsequently the Logo language associated with "Turtle Geometry" and more recently the Processing language particularly well suited to the computer exploration of shapes and colors. Regarding our robot Boustrophedon, it is built around the theory of "Arithmétique des formes". It leverages both computer algebra systems and symbolic / functional languages.

4) Generative Concrete Art

The approach that we put forward for producing Concrete artwork using a robot follows a sequenced conceptual framework we refer to as Generative Art by Synthetic Reasoning. As shown by the scheme on the left of Figure 3, this approach is very different from the process of producing artwork by what is called connectionist artificial intelligence [AI generative Art] schematized on the right of Figure 3. However, the Synthetic Reasoning approach, on the left of the figure, shares certain objectives with so-called symbolic artificial intelligence.

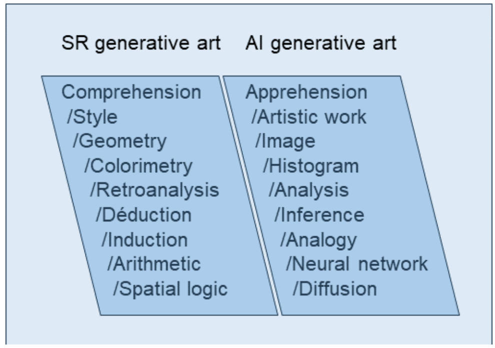

Figure 3. Synthetic Reasoning and Artificial Intelligence.

5) The three pillars of Concrete Art

5.1) The search for disorder

Disrupting or ordering geometry differently, deconstructing or structuring differently the rules of color composition are among the principles that fuel the creative process in Concrete art. To disorganize the geometry, or deconstruct the colorimetric rules, the robot must introduce an element of randomness while to order the geometry differently or structure the colorimetric rules differently, the robot must use combinatorics. The concept of randomness in Concrete art was developed by artists like François Morellet and Vera Molnar. "Vera Molnar introduces variations resulting not from her subjectivity but from random data, throwing dice, using telephone directories or tables of random numbers taken from math textbooks and computer programming” [4]. Technically, from our robot's point of view, we must distinguish pure non-deterministic randomness generated for example by physical source, from deterministic pseudo-randomness with a digital value or initial seed. Pseudo-randomness has a repetitive nature for a seed identical and is commonly obtained from computer programs. However, when teaching a robot the concept of disorder, we will use another approach that is much richer at the creative level: Chaos theory, which is particularly well suited for blurring an image and color scrambling. The non-random permutations obtained by swithcing from a first indexing system of one SFC type to a second indexing system of another SFC type will lead to perceptions of visual disorder (figure 4). The so-called SFC combinatorial curves are introduced in paragraph 6.2.

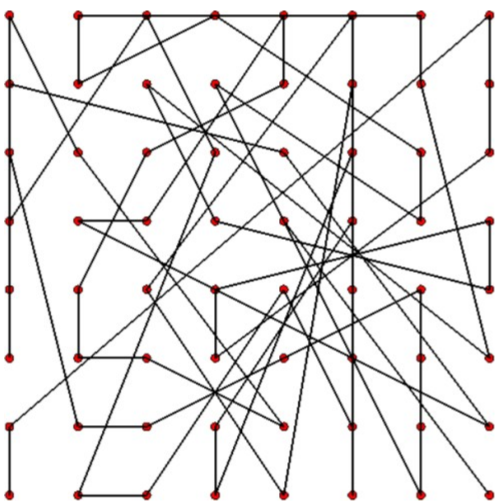

Figure 4. Disorder by SFC permutations

The robot will thus be able to reproduce the spirit of “Homage to Dürer, 225 accidental variations, direction chaos” by Vera Molnar from 1990 without any random variation.

5.2) Geometric partitioning

The theory of Tiles is probably one of the fields of mathematics closest to art. The number of combinations to partition the plane is enormous and the application of the underlying theory of groups of geometric transformations is easily programmed. The tiling theory is already observed in Islamic art before being revisited in the modern era by M.C. Escher. A combinatorial vision of tiling of the plane was introduced by Sébastien Truchet in 1702 with an eponymous theory. More recently, the theoretical and computer work of the classical geometer and structural engineer Janos Baracs in the 1980s [3] gives a second algorithmic breath to this theory, otherwise well known to artists of Concrete art. In 2015 Lorenzo Bocca published his “Sperimentare geometrie” which renewed the genre by exploring the partitioning of the plan from tiling freed from theoretical mathematical constraints.

Figure 5. Truchet tiling. (Source : Guillaume Pelletier-Auger 2016-2023)

5.3) Colorization of regions

The colorization of the regions resulting from a partitioning of the plane can be carried out by our robot using classic techniques, either by using random approaches, or by using graph theory and theorems such as that of The Four-Color or by using diffusion's methods known as "Flood Fill algorithm". In the case of our Boustrophedon robot, we chose a topological approach much faster and more powerful than the previous ones, making it possible to control the colorization of the plane by connecting it to logic.

6) Teaching reasoning to a robot

6.1) Arithmetic of forms

The Arithmetic of forms [7], is a formal system aimed at teaching geometry to a machine by converting all topological concepts in pure arithmetic. The classical topology pioneereed by mathematicians like Moebius, Listing and Poincaré, often called rubber sheet geometry, is extremely complex to computerize due to its heavy data structures. Topology also suffers from calculation precision problems, particularly when it is used to classify geometric shapes (e.g., homeomorphism) based on mathematical indicators such as Betti numbers.

To teach topology to a machine or robot, the Arithmetic of forms identifies and exploits the structural isomorphisms existing between the topology and the arithmetic. This approach makes it possible to eliminate all the main problems of computerization of the topology but also to control the arithmetic by logic, building upon pioneering work of Kurt Gödel [7]. For the process of arithmetization of calculations, Arithmetic of forms calculates algorithmically an integer value associated with each point in space. This value, called density, will subsequently be used in very different ways, either to make logical decisions or simply to assign a color to the point by computer hashing systems created from static or dynamic color tables. To enable the creative robot to determine and control arithmetic into a design interface, the Arithmetic of forms uses the theory of lambda calculus and recursive primitive functions, pillar theories of algorithmic computability. The creation phases are then as follows: our robot first generates polynomial forms with integer coefficients, forms that can be represented in two or three dimensions by a machine. Subsequently, the combinations of these primitive forms with logical operators chosen deliberately or randomly, allowing the robot to obtain visual and colored geometric results. Technically, the programming language learned by the robot belongs to the family lambda calculus interpreters such as the Scheme language or its modern algorithmic variants like the Julia language [8].

6.2) SFC metacurves

Figure 6. Vera Molnar, 2023 *Croix en lignes (source www.oniris.art)*

Space Filling Curves (SFC) are curves whose theory was defined around the 1900s by most well-known mathematicians like Cantor, Péano, Hilbert. These curves are universally used in many areas of computing, and, for our Boustrophedon robot, they form the heart of the process of generating chaotic partitions of the plane and chaotic generation of colors.

The theory of SFC was completed in 1973 by the mathematician Wunderlich, then extended in 2022 to Gray's metacurves as part of the development of a new kind of geometry called boustrophedonic geometry [9]. Gray's metacurves are curves that visit each point of regular and deformable grids in n-dimensional spaces only once. In the world of Concrete art, the most recent work (2023) by Vera Molnar (figure 6) explores so-called Péano's curves whose construction process can be traced back to the design of Hindustan seals before Jesus Christ.

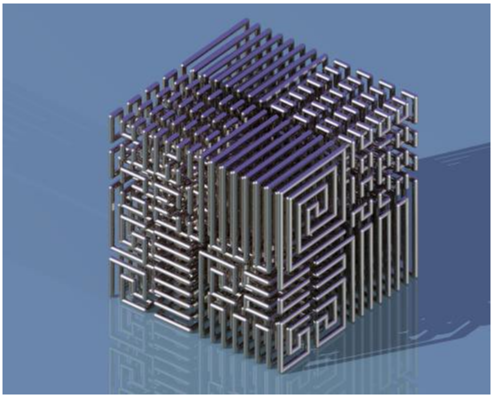

Figure 7. Gray's Meta curve in 3D

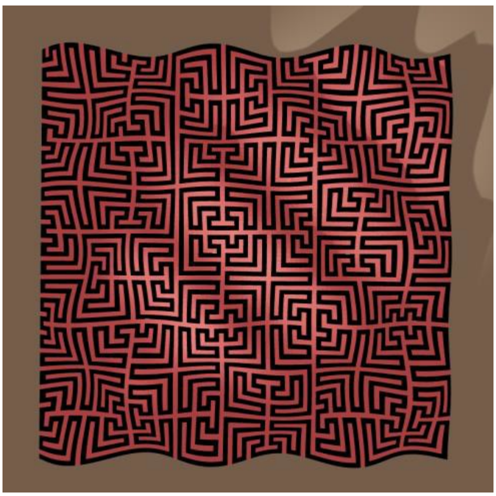

Figure 8. Deformation of metacurves

6.3) Topological stencils

To teach the robot to partition the plane, we instil in the robot a fundamental concept of topology: The Jordan's curve and Jordan's polygon. This concept is sufficiently powerful to cover a vast majority of cases of regular or irregular topological partitions encountered such as lattices, tiling, random polygons, SFCs and closed self-avoiding curves. Jordan curves or polygons have the property of dividing the plane into three elementary regions: the interior, the boundary and the exterior of the curve or polygon. The ternary coding {0,1,2} of the plane obtained algorithmically constitutes the first phase of the arithmetization of the plane. To describe Jordan curves, we will use implicit equations or piecewise parametric equations. In both cases the robot will operate in the projective plane to formalize the continuous deformations without special cases. The result obtained will be a partition of regions delimited by Jordan polygons, each region being associated with an integer numerical value: the density of the region.

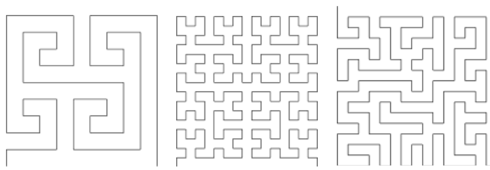

Figure 9. Jordan curves

6.4) The Péano's tiling

The Péano's tiling developed within the framework of boustrophedonic geometry [9] are directly controlled by so-called directrix of the Gray's metacurves which allow the robot to avoid the use of random or quasi-random methods in the generation process. The tiling generated are either chaotic or ordered in nature, the geometric order being ensured by the directrix of the metacurves.

6.5) Colors and attractors

To abandon any random approach regarding colors, the Boustrophedon robot is empowered with new capabilities allowing him to generate chaotic behavior from SFC attractors powered by so-called noise functions (figures 14-16). These attractors are distant cousins of the strange attractors which generate an apparent disorder from differential equations.

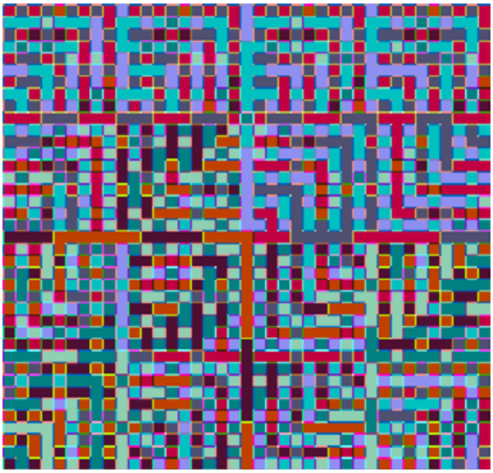

Figure 10. Generating a single-partition topological stencil

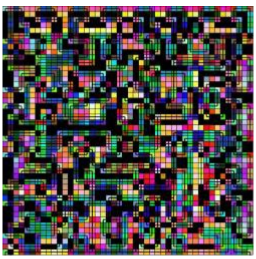

Figure 11. Generating a triple-partitioned topological stencil

7) Boustrophedon exhibition

This virtual exhibition features a selection of works produced by our Boustrophedon robot using plane partitioning schemes designed by Concrete art pioneers. These partitions are made from self-avoiding curves traversing regular grids and subsequently transformed into Jordan polygons which feed the topological calculations underlying synthetic reasoning.

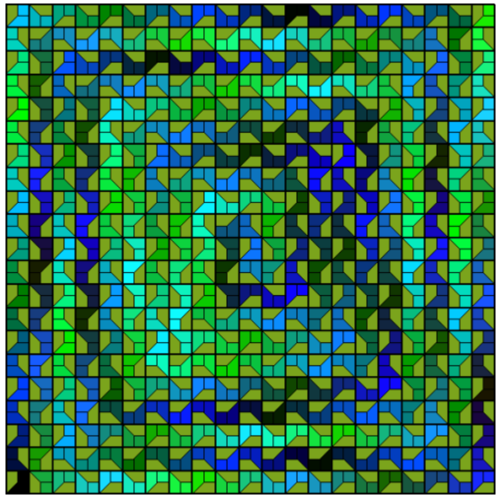

Figure 12 Chaotic colorization

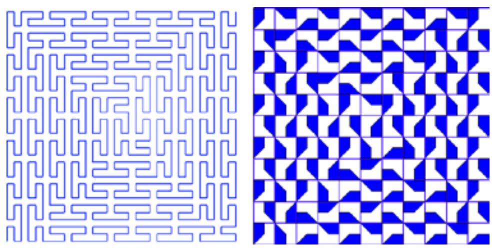

Figure 13. Ordered Péano tiling.

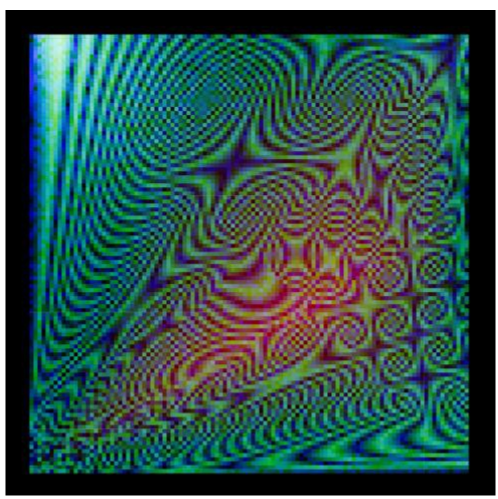

Figure 14. SFC attractor (Noise 1)

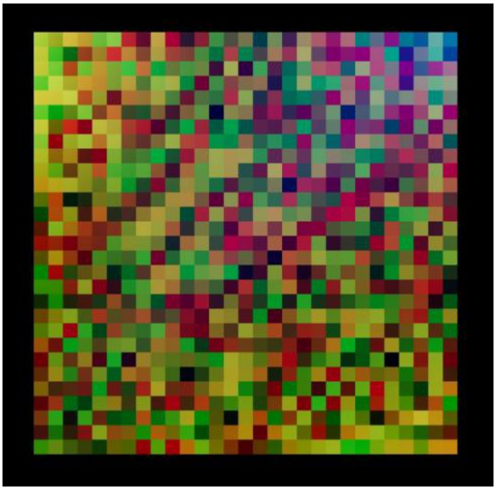

Figure 15. SFC attractor (Noise 2)

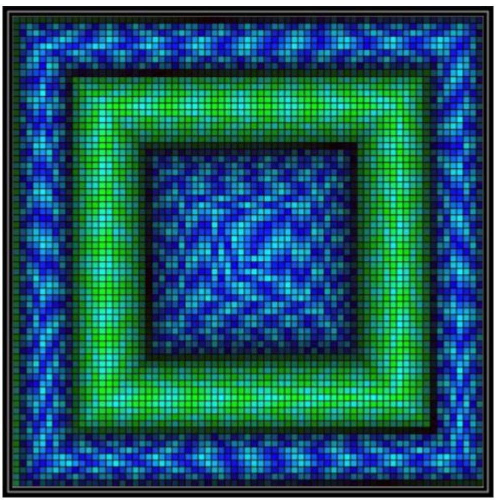

Figure 16. SFC attractor (Noise 3)

7.1) Rotor curves by Boustrophedon

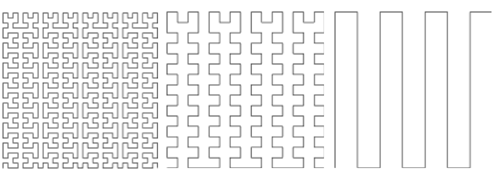

Figure 17. Rotor metacurves

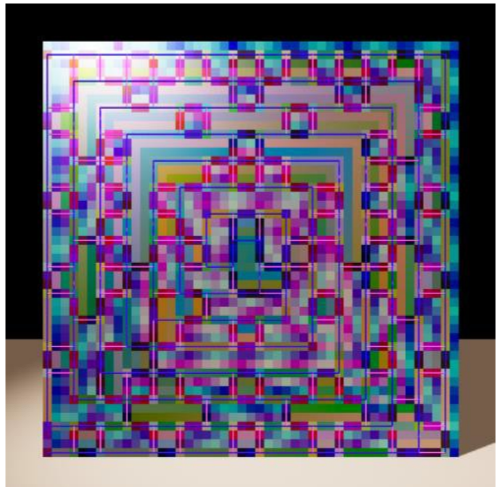

Figure 18. Topological stencil by superposition of partitioning curves

7.2) Waclaw Szpakowski' F13

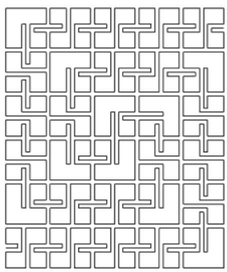

Figure 19. Retro analysis of the Szpakowski F13 self-avoidant curve1939-1943

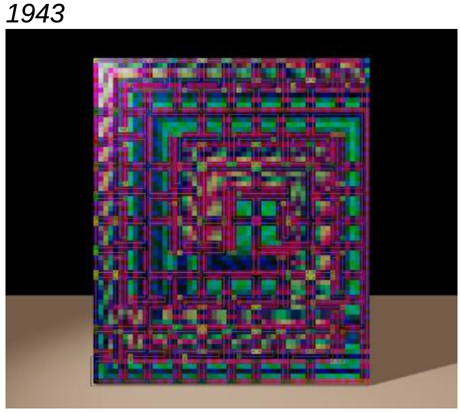

Figure 20. Topological stencil Szpakowski F13 1939-1943

7.3) Meanders by Anni Albers

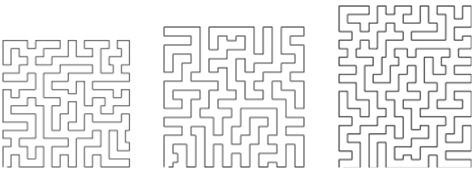

Figure 21. Retro-analysis of Anni Albers' Red, Yellow and Orange Meanders

The partition of the plane is made from the combination of a series of three meanders. The relative positioning of the meanders is obtained by playing in a non-random manner on the indices of the meander points.

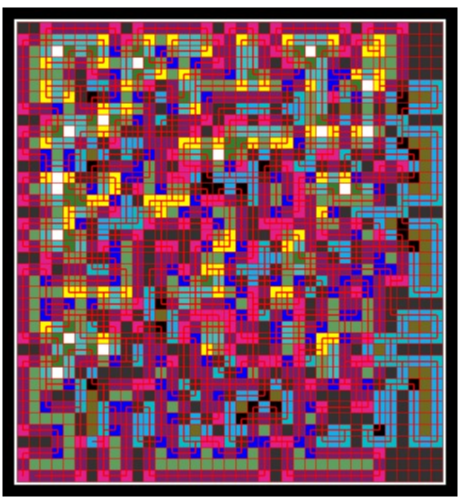

Figure 22. Topological stencil of the Meanders combination

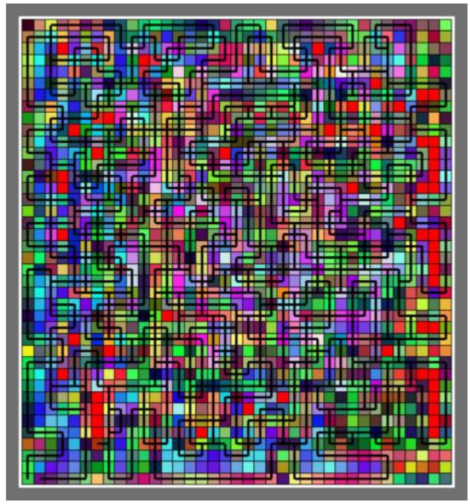

Figure 23. Topological stencil of the combination of Meanders with introduction of colorimetric chaos

7.4) Spiral by Julio Le Parc

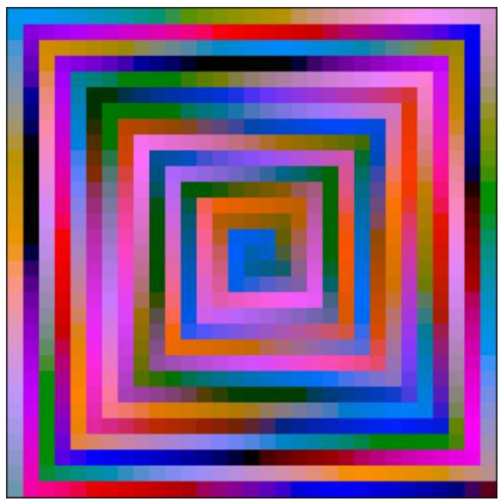

Figure 24. SFC color encoding and combinations

7.5) The Java of squares by Vera Molnar

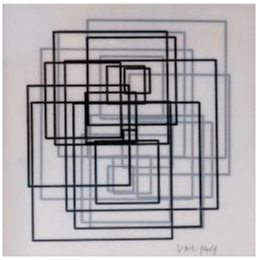

Figure 25. The Java of the 24 squares (courtesy Vera Molnar)

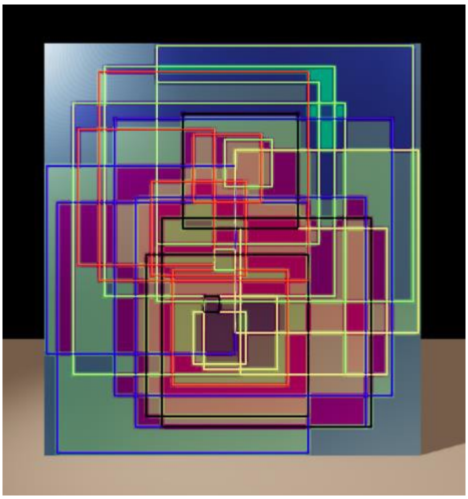

Figure 26 Topological colorization of Vera Molnar partitioning

7.6) The Majus effect by Victor Vasarely

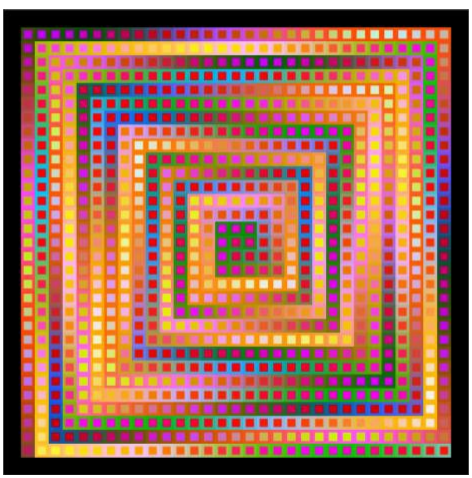

Figure 27. Restitution of the Majus effect by entanglement of metapixels.

8) References

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