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The science of networks: network graphs in urban navigation design problems – mobility, transportation systems, and movement paths generation

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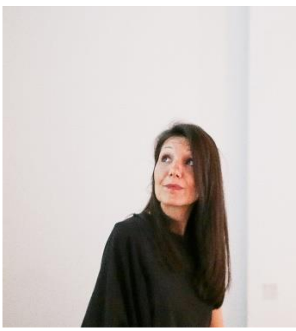


Abstract

Within the aim of testing generative computational design methods for design problems at the wider spatial, regional, and urban scale, a specific generative design task has been defined. It has been placed within the interdisciplinary field of urban planning, analytics, and design, computational and data sciences, as well as gaming and spatial experience and scenario construction, targeting particularly one of the territorial systems – mobility and transportation. Due to the fact that such infrastructure represents one of the most developed networks with constant internal and external growth, network science came as a supportive framework for bridging the gap between certain areas of spatial disciplines and data and artificial intelligence disciplines.

Generative design problem definition branched along the two tracks. The first one addressed the question of the design of the spatial system of urban and regional mobility, with special regard to its dynamics (growth and restructuring); the Grand Paris Express has been taken as a field of operation. The second addressed the subject of movement path generation within such system, according to the defined criteria, movement scenarios and objectives, and navigation and mobility parameters. The network and the moves within, could have been considered as both abstract and real-world environments and operations, susceptible to the research of both – gaming strategies (academic, operational, and heuristic gaming) and real-world problem-solving strategies.

In that respect, the study seeks the best set of computational design methods for solving defined generative problems, including graph methods, along with inquiries into modes of data representation and possible interoperability between various software frameworks. It discusses some of the steps taken in this direction, particularly their viability regarding the posed objectives.

Keywords: computational design methods, generative design problem, urban systems, urban computing, urban movement, network science, network graphs, movement path generation, smart and intelligent mobility, programmable city

1. Disciplinary, territorial, and programmatic scope

1.1. Inter- and cross-disciplinary alliances

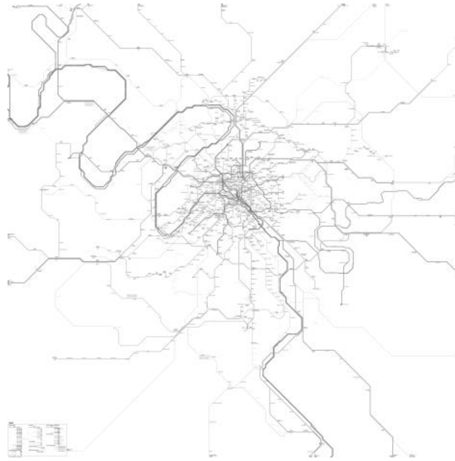
Several disciplines had to be called to attention considering a) the aim to test computational design methods for generative design problems at the wider spatial, regional, and urban scale, b) the chosen spatial subsystem (urban and regional mobility), and c) the complexity of the defined problem starting from this end of the spatial magnitude. Due to the fact that the main research subject is operated in space and includes issues at various scales, the overarching function has been assigned to the architectural and urban sciences. They are chosen for those responsible for comprehension and articulation of all the urban or regional subsystems, their concerted work, and built environment, as well as coordination of all the interested parties and powers that operate in space. The topic on infrastructures and urban and regional mobility implied involvement of traffic, civil, and mechanical engineering and sciences, along with specific contribution provided by operational and organisational sciences with their focus on command-and-control systems design with regards to traffic operation. Finally, a special place providing entrance to advanced modes of operation and automation, data acquisition, processing, and representation has been given to applied mathematics, or more precisely data and artificial intelligence sciences, including special insights in visual programming as a response to the visual representation request. In a convergent interpretation, the project has been situated within the field of urban computational planning, analytics, and design, targeting special contribution of data and artificial intelligence sciences to their new modes [1, 2, 3]. One branch of research placed specific interest in spatial decision-making and spatial experiences, or scenarios construction, including gaming as either a possible direction of the defined problem development or the medium of and environment for its investigation [4, 5].

1.2. Territorial-programmatic congruence: spatial scope and boundaries of defined mobility problem

Designing a generative problem required a precise contextualisation regarding the major research subject – a definition of its scope and scale in both spatial and programmatic terms.

As regards spatial scale, the problem has been defined by attributes of *regional* and *urban*, implying *city-region* as an operative territory and concept. In this particular case, the city of Paris - its metropolitan (greater) area and administrative domain – has been taken for analysis and defined in terms of the stated spatial properties and concepts. On the other side, the main computational subject, or a programme, targeted one specific urban-regional territorial system. This system referred to urban communications as the broadest category, within which the focus has been narrowed down to mobility and transportation, more precisely, rail traffic as the subcategory to be singled out. Regarding previously decided spatial determinants, the network of such scope could not have been one-sided (of one particular transportation type), but multimodal.

Based on the territorial congruence between the boundaries of the Île-de-France region, its historical and more contemporary metropolitan counterpoints, and the idea behind the *Grand Paris* (or Greater Paris) concept and strategy [6], along with both territorial and social aims that the planned communication infrastructure pursues, the chosen network-as-simulation-environment and its spatial scope, or range of influence, accorded with all of them (Fig. 1). This has also implied the network's magnitude, number of nodes, transportation types, and all lines of connection (edges). It has been decided not to parse its lines and increments by staying only within the smaller city area. Rather, the whole region has been considered, within which even provisional new nodes, whose liability had yet to be evaluated, comprised the network's integral part. These new nodes have been of specific importance regarding their role in optimisation tests and investigation of the network's dynamics (growth and restructuring).



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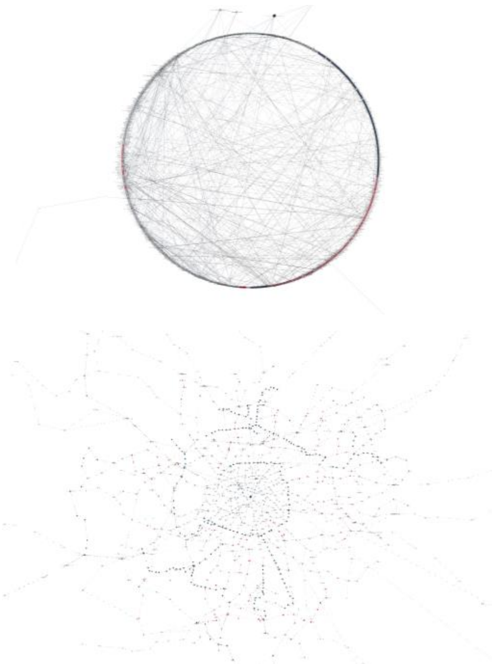


Figure 1. Correspondence between various aspects of the *Grand Paris* (Greater Paris) concept and designation (a, b, c) as references for deciding upon the range of the network to be designed (d, e).

a. Greater Paris rail line network; b. New metro network topological map; c. Grand Paris 2008 competition - MVRDV winning proposal; d. Grand Paris network - railway transportation system graph; type: undirected graph, Layout: Radial Layout Software: Gephi. Source: © Dragana Ciric, 2023
e. Paris railway transportation system graph; type: undirected graph, Geo Layout. Software: Gephi. Source © D. Ciric, 2023.

In applied tests and functional systematisation regarding the structure of algorithmic computational problem-solving method, the network represented a system of possible *moves* (regarding gaming terminology), or *spatial movements* (regarding physical mobility) within both abstract and real-world geometry, environment, or system of "legal actions". Legal actions have been bounded by the algorithmic "policy", which contains instructions for actions that can be performed, and thus be evaluated and executed.

2. Science of networks: network graphs for computational design

The *network science* [7, 8] comes as the next topic to be discussed. By constructing the main theoretical framework for the investigated generative design problem and broader computational problem-solving set of methods, it sets itself as a precondition for tests and application of each singular method. Well-analysed and observed in Barabási's major didactic volume [8], where it has been represented through structured units of a network science training programme with resources and designed student exercises, the science of networks has been made accessible to a wider range of disciplines. Putting upfront its topological model and scaffold for resolving problems based on dynamic relations and contingency between the investigated elements, it became applicable to various situations. The mathematical formalism of network science and the network properties such as those of being driven by empirical data and having quantitative, mathematical, and computational nature [8], make networks and network graphs suitable for investigating many types of systems. Spatial systems represent only a part of them. With respect to the chosen research subject and content, the network that could have been applied in this particular case directly corresponded to the existing spatial configuration of targeted elements (urban mobility infrastructure and network elements, comprising stations-nodes and lines-edges; Fig.3a), though it could have been represented differently (Fig. 2, 3), given the form of a network graph. The other, nonspatial application of such relational system, has been found in problem-solving methodology analysis and design at two levels – firstly, with regards to the complete set of problem-solving methods, or computational problem-solving methodology, and secondly, at a lower level, regarding each method and a specific problem it aims to solve, as a part of the decomposed problem-solving structure (Fig. 4 and 14a). At these levels, network graphs are operating as process propagation graphs or diagrams, also known as *flow charts* – they represent design, or problem-solving procedure graphs that correspond to the logical sequences of (computational) decision-making steps and/or problem-solving set of methods.

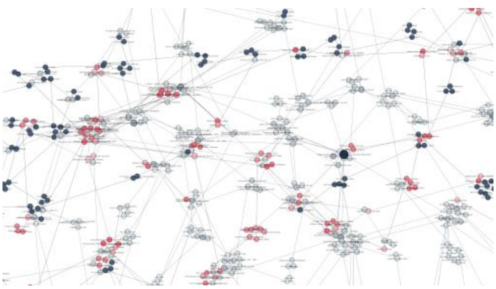


Figure 2. The fragment of the Paris railway transportation system graph; type: undirected graph, OpenOrd Layout. Software: Gephi. Source © D. Ciric, 2023.

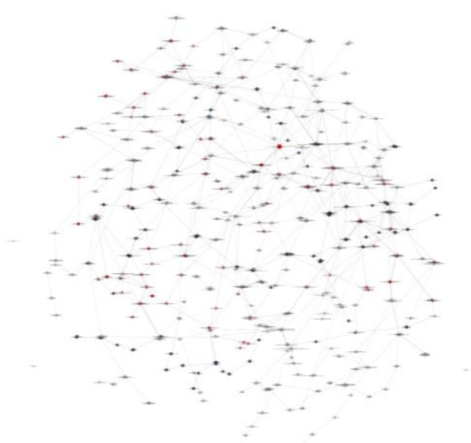
Network graphs and graph operations will be described next. While networks are related to the *science of networks*, graph definition comes from the *graph theory*, the branch of mathematics that grew out of Euler's proof [8]. To represent the network as a graph implies its definition in terms of graph theory, or through graph-theoretic formalism and language (graph, vertices, edges, degrees, degree distributions, paths, distances, types (weighted or unweighted, directed or undirected and bipartite networks, etc.). Graphs describe how components (nodes and vertices) interact with each other (links, edges) through *a kind of a map of its wiring diagram* [8]. As common formalisms, networks usually refer to real systems, while graphs are used when referring to the mathematical representation of these networks [8, p. 8 (ch. 2)]. The advantage of graphs for this particular design problem, and in general, lies in their *property to make certain problems become simpler and more tractable* [8, p. 3 (ch. 2)] and express paths which are rather inherent to graphs than the result of some kind of external finding [8] (they are already inscribed in graph concept, functioning, and formal representation). Stated properties encode graph structures, and therefore limit and enhance their behaviour.

In the case of the following generative design problem, regarding its first part – network construction – two types of graphs and networks have been analysed. The real network is sparsed and it has been created according to the real situation (Fig. 3). The abstract network has been represented by the complete graph, intended to be tested in Grasshopper (Fig. 6). The latter network case has been considered as a provisional geometry for defining optimum problem-solving methods, whose composition could eventually be applied to both sparse and complete graphs. Also, graphs have regularly been used in computational problem-solving methods in Grasshopper, and in the process of defining the complete problem-solving methodology, or a set of methods and actions to be taken towards the defined generative design aims [9]. As explained earlier, these graphs have been used to trace the logic of computation problem-solving design methods concatenation and to propagate generative design procedure.

2.1. Generative design problem definition

Generative design problem definition branched along the two tracks. The first one addressed the question of the design of the urban and regional mobility spatial system, with special regard to its dynamics (growth and restructuring). The Grand Paris rail system has been taken as a field of operation (Fig. 3 and 6) including all transportation types – metro, tram, regional express, and speed rail.

Figure 3a. Paris railway transportation system graph; type: undirected graph, Geo Layout. Each node has been assigned its geographic coordinates – longitude and latitude. Software: Gephi. Source: © Dragana Ćirić, 2023.



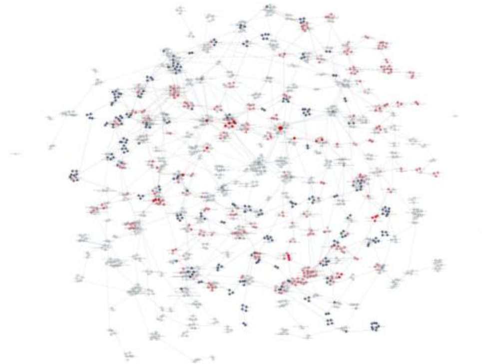


Figure 3b. Paris railway transportation system graph; type: undirected graph, OpenOrd Layout, Software: Gephi. Source: © Dragana Ćirić, 2023.

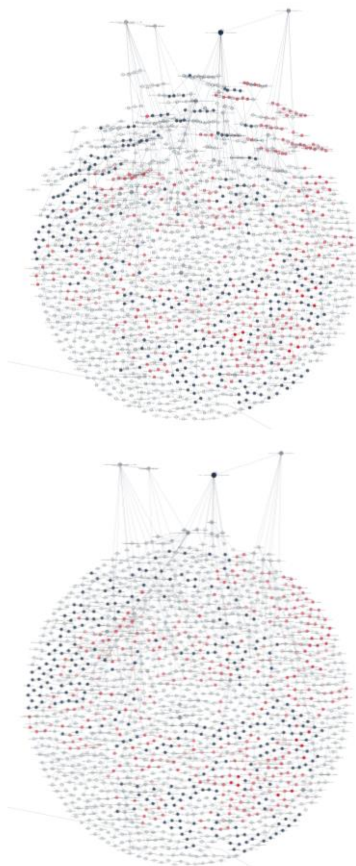
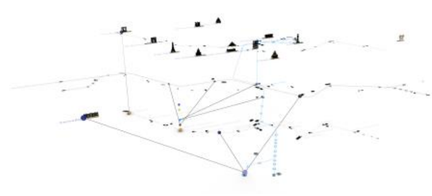


Figure 3c. Paris railway transportation system graph; type: undirected graph, Fruchterman-Reingold Layout. Software: Gephi. Source: © Dragana Ćirić, 2023.

The second addressed the movement path generation problem within such system, according to the defined criteria, navigation and mobility parameters, and specified agents with their path preferences, movement scenarios, and objectives. The problem has been defined as follows:

Generate an urban movement path of an agent along the lines of the city rail transportation system while being guided by relations between the states/points within its network (distance, proximity, thematic character, frequency, etc.) and landmark class of destinations (Fig. 4). Define parameters for path character, movement objectives, constraints, policies, and movement agendas. Suggest path increment options in each state and continue the process after a decision has been made (a new state has been occupied) with feedback propagation to be included in statistical analysis and to improve new playouts.



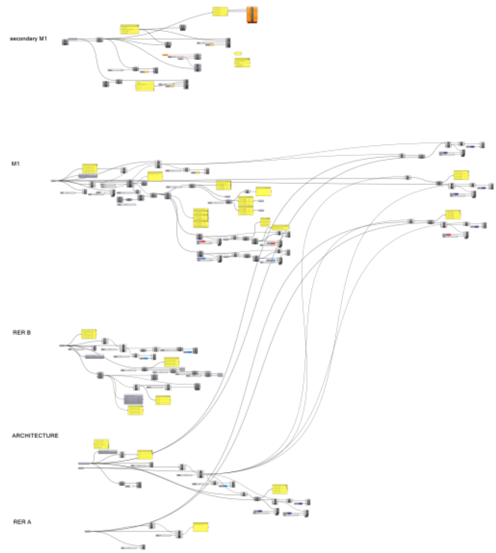


Figure 4. Simulation of the dynamic movement along the exemplary metro line, while the two closest points on the intersecting or marked metro lines and the closest landmark destination are displayed as subsequent points (states) to be occupied as a part of the function suggesting possible actions. Chosen stills of an animation render simultaneous movement of two agents along the two lines, while variable number of closest landmarks and closest transit stations to be displayed can be programmed (the stills render one landmark and two closest intersecting connections for each agent). Source © D. Ciric, 2023. https://dciricnetworks.tumblr.com/, https://64.media.tumblr.com/036b9b81066623f82155223ecc171a50/a1edba9043f32fba-da/s1280x1920/f0e5572a01d8dd2d6fbd18de72adb1c98de3da8a.gifv, https://64.media.tumblr.com/704220f63d4a09c895c58eef54c9ebf7/da42d815e8d6c12a-a1/s1280x1920/e49a34a05b923244e5e8004b6bed3aa4b9d5f66d.gifv

2.2. Computational problem-solving set of methods - sequential tasks

Following a twofold design problem definition, the set of computational methods investigating and offering probable solutions (examining solutions distribution space) has been established [9, 10]. Some of the previous studies documented their overarching problem-solving methodology in more detail [9]

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and by constructing the methodology for the case analysed here, they also provided a framework to be applied in other similar situations. While it is better explained in stated sources [9, 10], for the purposes of this paper the methodology has been decomposed and parsed to singular exercises whose computation will eventually become integrated into the final set of methods for network system design, analysis and optimisation, and movement path generation, as indicated by the problem definition.

2.3. Software environment

The question of proper architectural and network design software for required computational problem-solving methodology ensues. The considered software group comprised Rhinoceros–Grasshopper, Generative AI Autodesk, and esri ArcGIS or ProGIS software packages. They had to be leveraged in terms of the computational design methods that would be combined and supported by their functions.

A smaller analysis and algorithm of the procedure “how to choose proper software and functions” according to the desired environment and objectives have been made (Fig. 5).

Figure 5. The linear dendrograms with a tree structure/layout represent the way to decide about the environment for the defined design problem’s testing and how to choose the proper computational method and software for resolving specific issues within the chosen environment. The algorithm considers 1) real-world geometry and situation along one branch and 2. abstract geometry along the other. The continuing branches/categories contain elements necessary for the modelling process and explain models that are used further in the design process, information systems and data formats used for its modelling, ending with plug-ins and software in which devised operations can be performed and stated data formats processed or made operable. Source: © Dragana Ćirić, 2023

Regarding previous analysis of algorithmic, parametric, generative, and AI methods as methods that all can be applied in hybrid computational problem-solving methodological sequences [9], Rhinoceros-Grasshoper visual programming environment has been selected as a starting point, while advanced AI approaches have been assigned to experts [11, 12, 13] to be performed in later phases, once the first decisions and analyses are finished and adequate design problem definition approved. Architects usually combine a larger number of software products for different design tasks. The most efficient process, however, would be executed by a single software, or by a software capable of executing the majority of required operations, usually those belonging to the same functional cluster (e.g. design conceptualisation, research, drawing and modelling, or visualisation and rendering).

With respect to problem-solving aims and defined methodology, Grasshopper appeared as the most functional for the first trials, particularly regarding plug-ins specified for some of the urban computing phases (e.g. Urbano). Prior to its complete use, several software products for network graphs have been tested as well (Gephi, Cytoscape). Even though the process in Grasshopper eventually had to be repeated, the outputs made in Gephi – e.g. the urban rail system graph of Grand Paris Express (Fig. 3) – had a significant role in resolving phases of data acquisition (analysis of relevant data and sources), selection (decision on territorial and connectivity scope, including types of transportation), and ordering (creation of lists suitable for targeted graph operations in the available software).

2.4. Parametrisation

Further consideration was given to the fact that some of the features, in order to be represented dynamically and adopt various values, needed to be determined as variables and to be parametrised in line with the taxonomy of parameters [14] and broad literature on parametric design [15, 16, 17]. These features included the following: location (nodes within the designed network, Fig. 3), programme (Fig. 6), transportation type (Fig. 7), transportation lines with their determinants, including passenger frequency and connections (Fig. 8), and network specificities that will provide distances for proximity operations (Fig. 9)

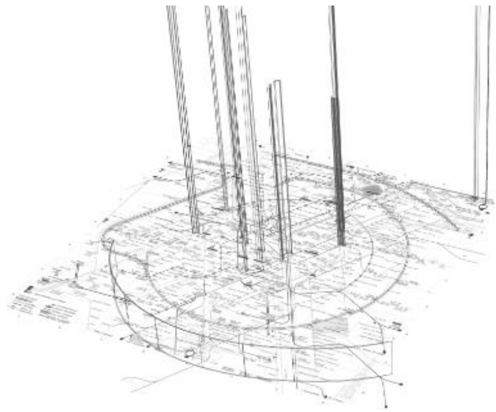


Figure 6. New topological Paris metro map showing extraction of cultural landmarks as a specific programmatic layer to be used in different movement path scenarios and agents' objectives regarding chosen destinations. Source: © Dragana Ćirić, 2023.

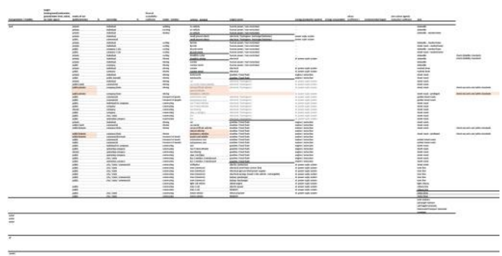


Figure 7. The table containing all transportation types (focused on land transportation, while only mentioning air and water transportation), explained through the following categories (attributes): modes of use (public/private), ownership (private/individual, private-company/commercial, public-commercial/company, public-State), financial accessibility coefficient, vehicle type and vehicles' details, engine power, energy (production system), energy consumption, zero-carbon agenda evaluation coefficient, area of occupation, safety coefficient. Source: © Dragana Ćirić, 2023.

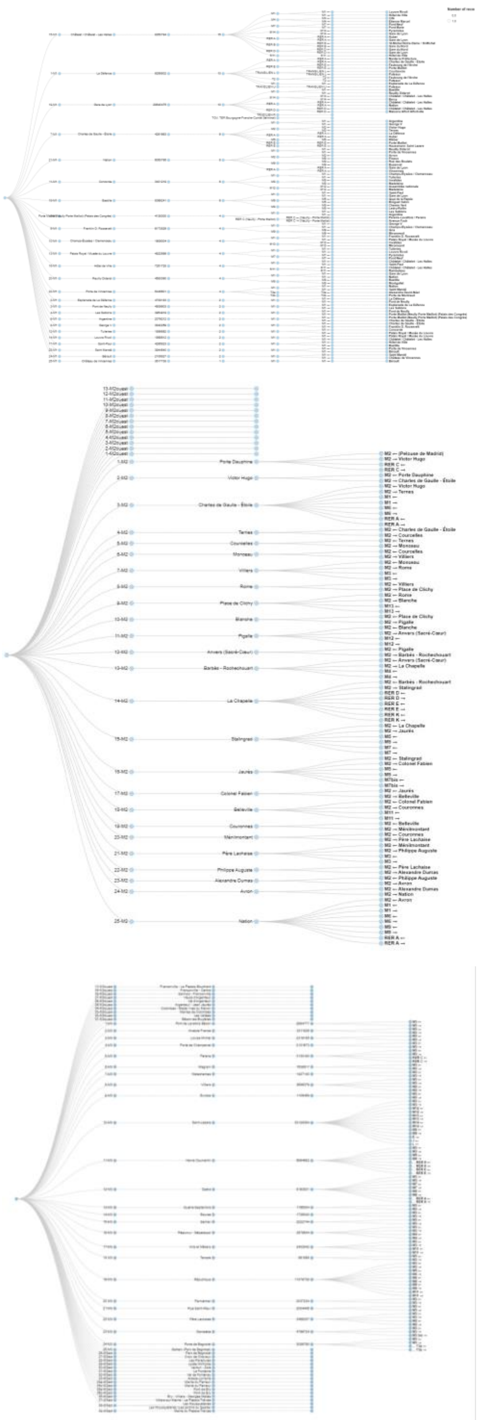


Figure 8. Definition of each transportation line – the examples of metro lines M1, M2, and M3 respectively, with the ordering label for each station, station name, daily/yearly number of passengers (frequency), node weight (number of connections, or branching lines), connections labeled by the type (M, RER, Transilien, tram, TGV, etc.) including directions, and precise connection label, or ID of each direction. Source: © Dragana Ćirić, 2023.

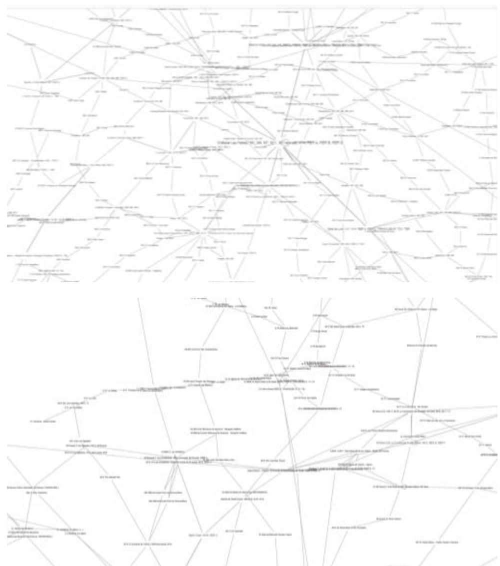


Figure 9. Gephi Geolayout of the Grand Paris network, displaying connection lines and names of each station only, implying therfore correct distances for the proximity search function. Source: © Dragana Ćirić, 2023.

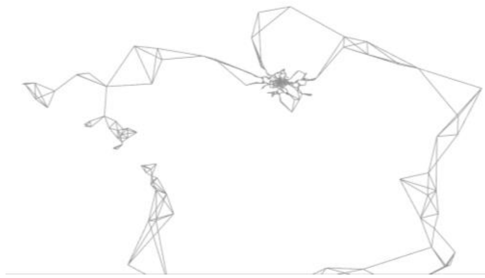
3. Problem decomposition, singular problem-solving computational methods and their smaller strings

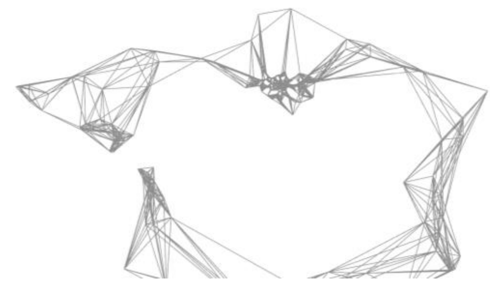
The final section will demonstrate some of the problem-solving methods made in Rhinoceros-Grasshopper. The main design problem has been decomposed according to the sequential design of its elements and functions related to them, and the structure of the section will follow problem-solving logic as stated previously and in referenced works [9]. The main division follows the bipartite nature of the generative design problem definition, explaining within each part the separate operations performed with respect to problem-solving methodology and final solution. Considering that not all of the solutions have been finished and refined, the section will focus on elementary ones whose combinations and concerted configuration lead toward the desired outcomes and problem-solving outputs at the local level. The targeted scientific feature is a representation of the advantages of problem decomposition as an inherent property of computational thinking, decision-making, and problem-solving.

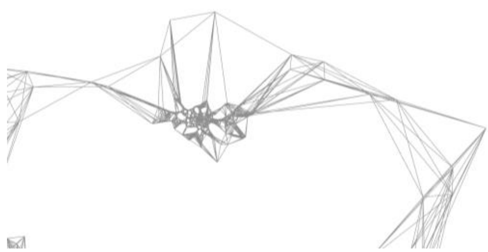
3.1. Network

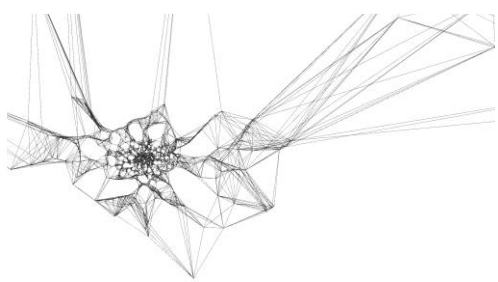
The first part of the design problem is network design or generation with respect to its dynamics. The exercises (also subsections) represent the following: 1) network design method (Fig. 10), taking into account the possibility of creating also a complete network for an abstract gaming environment, 2) localisation method (Fig. 11) – construction of the set of functions to define spatial position, or location within the network, with a likelihood of its change (the location is parametrised), 3) location search method (Fig. 12) – the search method that, as per defined criteria, analyses subsequent movements given a state/position within the network; the three exercises imply the final exercise as their concerted result 4) composition of localisation and search methods with respect to the designed network (composition of 2) and 3) with respect to 1)).

3.1.1. Network design









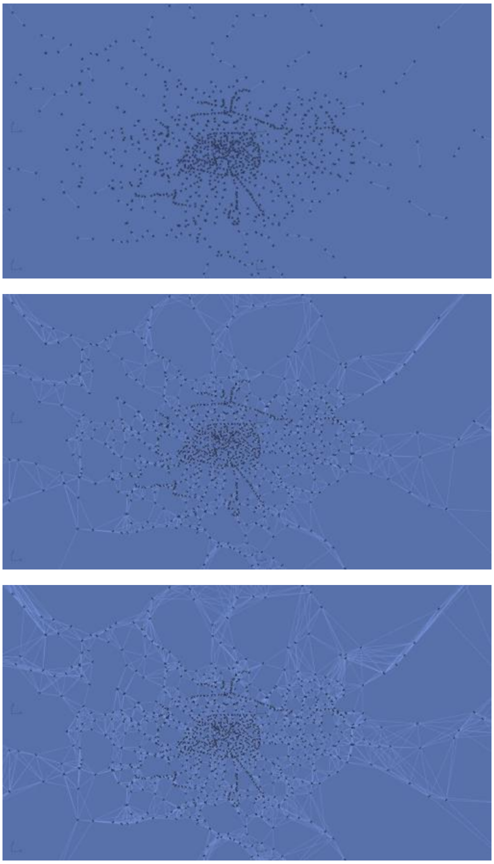


Figure 10. Network design. Software: Grasshopper. Source: © Dragana Ćirić, 2023.

3.1.2. Localisation

Figure 11. Localisation function (f_l), given the network (N) defined by nodes (S, ((S_0, S_1, ... S_n))). Grasshopper. Source: © D. Ćirić, 2023.

3.1.3 Parametrised location search: suggestions of possible moves, or states to be occupied within the network, calculated according to the defined criteria (e.g. distance/proximity)

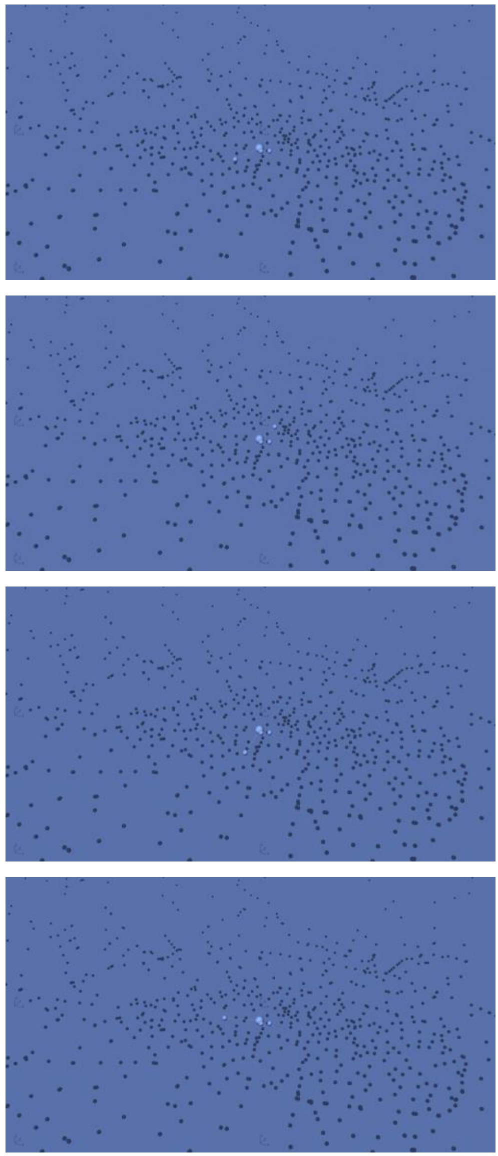


Figure 12a. Search function (f_s), given a state or a location (S_0) within the network as a set of all possible states (S),

displaying states/nodes to be occupied in a movement sequence S_A (S_1, S_2, ... S_n) performed from the designated/given state (S_0). Software: Grasshopper. Source: © Dragana Ćirić, 2023. https://va.media.tumblr.com/tumblr_s1edpid9fB1a96rc8_r1_720.mp4

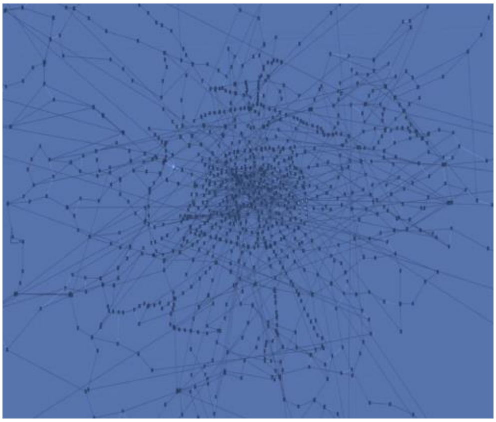


Figure 12b. The search function (f_s) displaying states/nodes to be occupied in a corresponding movement sequence, performed in a given state which has been parametrised to enable multiple starting positions as well as to be applied in a sequence of operations which define a movement path (location has been parametrised alongside the set of options resulting from the search operation). Software: Grasshopper. Source: © Dragana Ćirić, 2023. https://va.media.tumblr.com/tumblr_s1earwoG1t1a96rc8_720.mp4

3.2. Movement path generation

Movement path generation combines several previously designed computational methods as problem-solutions: a) localisation/positioning function with respect to the given network, b) search function, c) parametric control of variable values/entities (type of the destination, number of levels to be searched), and 4) decision-making regarding the state that will be occupied. Their composition and proper parametric control enabled movement path generation as demonstrated in the following subsections and through linked animations whose stills have been represented to illustrate performed operations (Fig, 13 and 14).

3.2.1. Urban movement path generation with fixed decision-making criterion and its value (same iteration process)

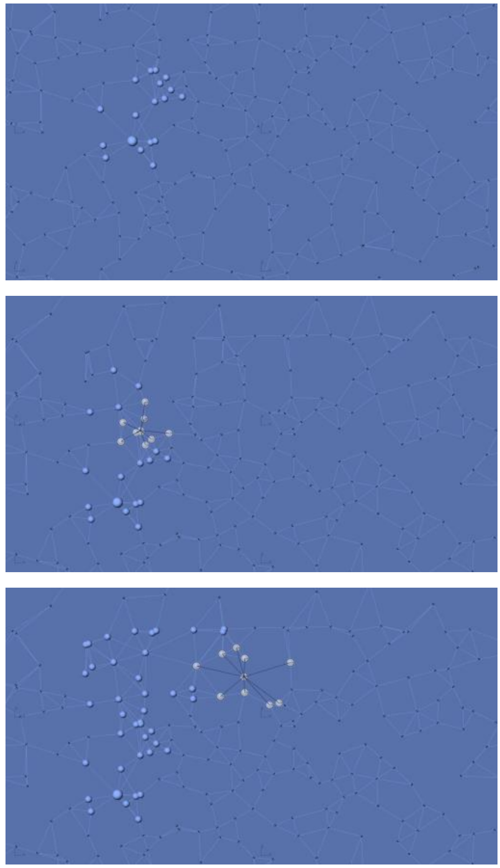


Figure 13. Movement path generation based on proximity search function, analysis, and decision-making in each increment point with regard to the defined variable parameter. Software: Grasshopper. Source: © Dragana Ćirić, 2023. https://va.media.tumblr.com/tumblr_s1e92yJX8r1a96rc8_720.mp4

3.2.2. Urban movement path generation with parametrically controlled and moderated decision-making (variable iterative process of decision-making)

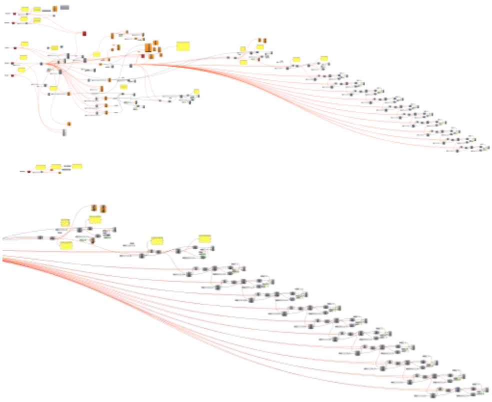
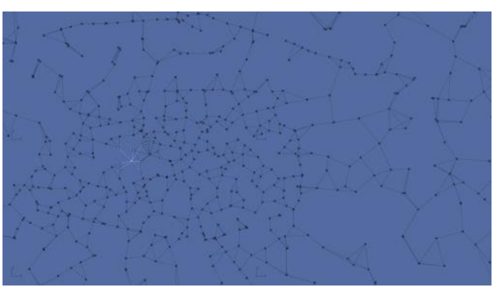


Figure 14a. Propagation graph of sequential passes from one state to another based on decisions made in each state, given a range of possible moves within the system from a designated state. The search function performed in each state, from which the movement streams, is the same, while decision is parametrised, made and calculated differently in each iteration, implying the case in which the agent opts for a different parameter value - the state/node to be occupied differs with each move decision, as defined by the formula and the chosen criterion. The movement path (including traces of possible movement distribution) is generated, suggested (displayed) and preserved within the given software environment. Software: Grasshopper, Source © D. Ciric, 2023.



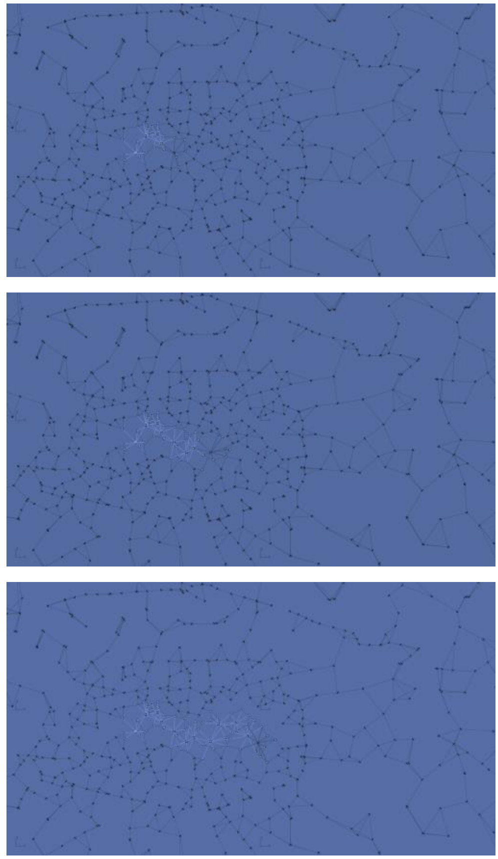


Figure 14b. Rhinoceros interface - Sequential passes from one state to another based on decisions made in each state, given a range of possible moves within the system from a designated state/network node (each state occupied based on performed decision-making). The generated movement sequences compose the complete trace of the movement path including all search results and proposals from the occupied states/locations. The overall policy of the exemplary path required decision-making according to the main objective of making the largest possible area coverage with the generated path (other policies would have caused and influenced different path results). Software: Grasshopper, Source © D. Ciric, 2023. https://va.media.tumblr.com/tumblr_s1fik2uIPa1a96rc8_720.mp4

4. Conclusion

The paper addressed a specific application of networks and network graphs in computational generative design problem-solving at the scale of the urban region. Considering the existing literature in the defined field with regards to both architecture and urbanism [18, 19, 20, 21, etc.], along with the objective of widening the scope of possible solutions and their diversification and variability [22, pp. 150-153], a specific focus has been placed on urban-regional systems planning, analytics, and design, with additional sources included [1, 2, 3, 4, 5]. The greater Paris administrative area and rail transportation network have been taken as research subjects to which the planned studies and demonstrations have been applied. The network was constructed using several software products, and defined tasks were performed within such simulation environment. Exercises for a decomposed problem-solving methodology and set of methods have also been provided – designed and tested – and their operation demonstrated to give evidence and arguments of the benefits and importance of the explained computational, algorithmic, generative, and AI modes of thinking and their application in both abstract and real-world spatial problem-solving [23].

5. Supplementary material

Network operations (test).
https://dciricnetworks.tumblr.com/
Source: Dragana Ćirić, 2023.

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7.

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