The generate method of Multi-storey Chinese Pagodas

Tang Zhong Senior Engineer
Zhang Yijie Doctorand
Tongji University, Shanghai, China, tangzzzk@online.sh.cn

Abstract: As many traditional Chinese architecture, pagodas are also very mature and formal. In this paper, we try to find the rule of the shape of Multi-storey Chinese Pagodas, and try to describe the rule in logical math language. This study will help computer programmer create the model of the pagoda in parameter-driven way. We wish more scholars were interested in study traditional Chinese architecture by generate method.

Keywords: Generate, Pagoda, Chinese, Math

The introduction of Chinese pagoda

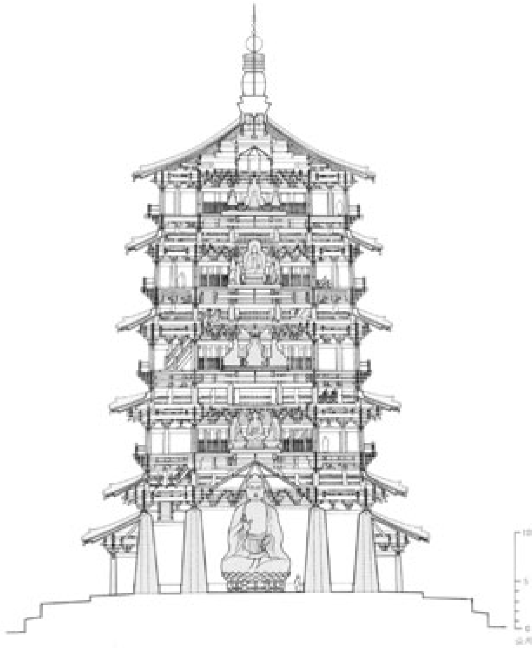
The word pagoda denotes a tower-like building constructed either of marble, stone, glazed and unglazed bricks, wood, iron or bronze. It is generally understood that the influence of the Indian Stupa was the origin of these buildings. Since a considerable intercourse between Indian and China was kept up by common Buddhist religious interests, there can be no doubt that certain types of pagodas at any rate are of Indian origin.

Figures 1: The Indian Stupa

It is not impossible that the real origin of pagodas was just a simple rectangular building with an additional storey added for effect and then perhaps more added until a tower-like shape was achieved.

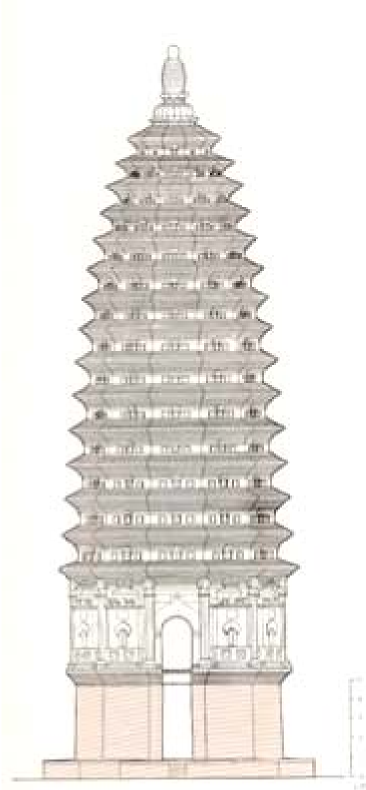
In the T'ang period (A.D.618–906) pagodas were usually simple, square structures; they later became more elaborate in shape and adornment.

Nearly all types of pagoda are now octagonal in plan, a natural evolution from the square when designing a tower. It should be noticed that pagodas always have an odd number of roofs in accordance with the Chinese theory of numbers. Each storey diminishing in which while simultaneously each order is reduced in height.
Pagoda is also the main integrating part of the Buddhist architecture, with varied styles and strong local flavours. Pagoda followed Buddhism into China around the first century, and developed into pavilion-like pagoda on which one can view scenery after immediate combination with traditional Chinese architecture.
Most Chinese pagodas are multistoried ones. Early pagodas were usually wooden and had quadrangle, hexangle, ocatagonal and twelve sided ichnographies. During the Sui and Tang dynasties, pagodas tended to be stone and brick. In the Liao Dynasty, solid pagoda appeared. After, in the Song, Liao and Jin dynasties, flower pagodas were introduced which were decorated with assorted carved flowers, honeycombed shrines, animals and Buddha and disciple sculptures, looked like flowers. Generally speaking, pagodas became more and more decorative.
The main reasons early pagodas in China had many storeys were, first, since pagodas were originally built to preserve Buddhist relics, which were considered the most sacred objects in the world, representing Buddha, they should be majestic and striking in style. Second, multistoreyed buildings were traditionally used by the ruling class to show off its power and wealth; they were also believed to be the residences of the immortals; therefore they were most suitable for enshrining the mysterious Buddha, the highest saint among the immortals. Third, high buildings of many storeys were usually awe inspiring and mysterious looking.
Chinese pagodas belong to several categories. Based on their style of construction, they can roughly be classified into four categories
First, is the multi-story pagoda. It resembles a multi-story tower with protruding up-turned eaves. The oldest and tallest of this type is a magnificent Yingxian Wooden Pagoda. Built in 1056(in Liao Dynasty) extant oldest and highest wooden Buddhist Pagoda is located in Yingxian County in Shanxi Province. The Pagoda consumed at least 3,500 cubic meters of wood. The pagoda is octagonal in shape, presenting an outward appearance of a five-storey and six-eave building, but actually it is nine-storeyed. Not even a single nail was used and the whole structure is weathered over 900 years of wind and storm and remained intact despite many strong earthquakes. It is indeed a masterpiece of ancient Chinese architecture.



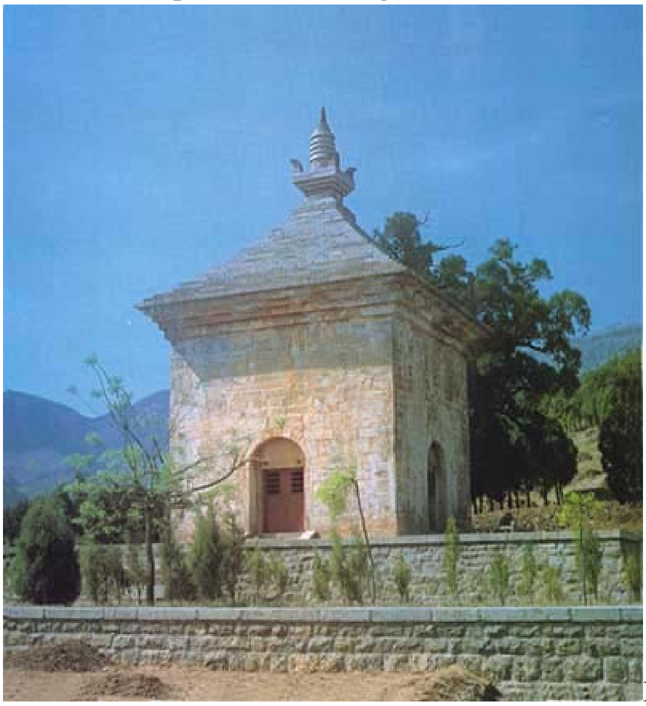
Figures 2: Yingxian Wooden Pagoda

Another type is the Miyanta. It takes its name from the many tiers of closely-set eaves at the top. Most pagodas of this sort are built of brick and stone. They are without doors or windows, except for holes that let in light. The earliest example is at the Songyue Temple on Mt.Songshan in Henan Province. Built in 520, it stands 40 meters high and has 12 sides capped by 15 tiers of eaves. (Figures 2)



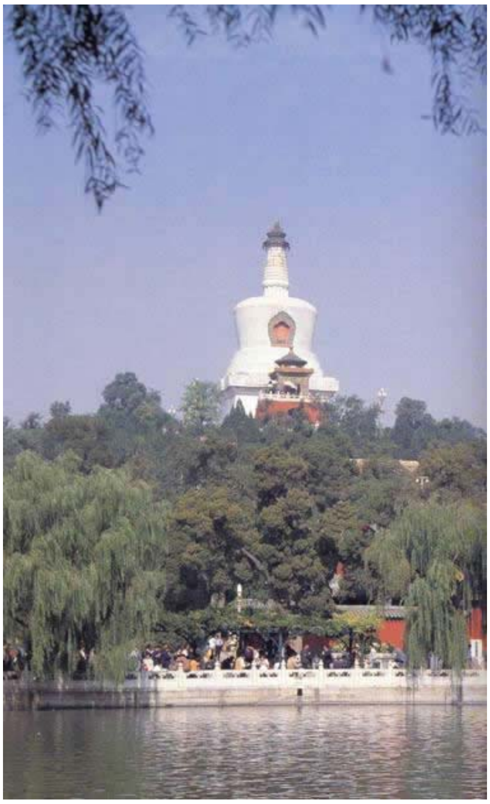
Figures 3: Songyue Temple Pagoda

The third style is the pavilion-style or one-story pagoda. Most of them were used as tombs for abbots and other high-ranking monks. The earliest of these still extant is the 1,400-year-old Simen Pagoda at the Shentong Temple in Shandong Province, East China. It is square, with a single roof and one door on each side.



Figures 4: Simen Pagoda

The fourth is the Lamaist style. Similar to Indian dagobas, the pagoda is a dome-shaped structure set on a large pyramidal platform. One famous example is the White Dagoba in Beijing's Beijing's Beihai Park.



Figures 5: White Dagoba

Ancient Chinese pagodas may stand alone or in groups. The largest group of pagodas is the Pagoda Forest at the famous Shaolin Temple in Henan Province. This group is composed of 220 brick and stone structures.

Figures 6: Pagoda Forest

Another well-known example is the unique Bamboo-shoot Pagoda in Yunnan Province. It consists of a central pagoda in the shape of a lotus flower, surrounded by eight smaller ones. From a distance the group suggests a bamboo thicket.

Figures 7: Bamboo-shoot Pagoda

Chinese pagodas may be square, polygonal or circular with each story separated by projecting roofs or eaves. A typical pagoda has four main elements: the underground hall, the platform, the body and the steeple.
The underground hall usually housed sacred relics, books and paintings. This underground palace was similar to the underground palaces of the mausoleums of emperors and kings in ancient China, but it was usually much smaller and contained

fewer funerary objects. The most important thing in an underground palace of a pagoda is a stone container with layer upon layer of cases made of stone, gold, silver, jade and other materials. The innermost case contains the Buddhist relics. The funerary objects in the palace may include copies of Buddhist scriptures and statues of Buddha. Underground palaces were usually built of brick and stone in square, hexagonal, octagonal or round shapes. Occasionally such a structure was built inside the pagoda or semi-underground.

The platform may be a simple structure, or it may be elaborately decorated. It supports the whole superstructure. In early times most pagodas had relatively low bases. Some bases are only ten or twenty centimeters high. They soon become indistinct and even unrecognizable from the ground after being damaged over the years. During the Tang Dynasty, in order to make pagodas such as the Big and Small Wild Goose Pagodas in Xi'an look magnificent, huge bases were built under them. Large bases were also added to pavilion-style pagodas during the Tang Dynasty, for example, the Pagoda of Monk Fanzhou in Anyi of Shanxi Province and the Dragon and Tiger Pagoda at Shentong Temple in Licheng near Jinan.

The shaft or the main part of the pagoda may be either solid or hollow. Solid pagodas are filled with bricks, stones or rammed earth. Occasionally, a wooden framework is installed inside a solid pagoda to strengthen the bearing capacity of outreaching parts of the pagoda. A spiral stairway sometimes leads up through this central shaft. Images of the Buddha are usually carved on the outside walls. Pagodas' roofs are often crowned with ornate carvings or studded with jewels.

Every pagoda is surmounted by a steeple, sometimes pointed and sometimes ball-shaped. They vary greatly in style and building materials. The most commonly used building materials for steeples are bricks, stones and metals. The steeple, as the tallest part of the pagoda, is extremely important. In Chinese it is called Cha, meaning land or territory representing "the country of Buddha."

The generation of the body shape of multi-storey pagodas

Pagodas were the delight of Chinese landscape-architects, and were located in variety of different situations. They were often in early times built adjacent to a temple or monastery as works of religious enthusiasm. However, the Chinese were very fond of erecting multi-storey pagodas on hill tops or mountain slopes, and they were placed so that they were axial with the entrances to a temple or place, and could be seen through the main gateway at the end of the vista.

As the multi-storey pagoda is tall enough to change the skyline of a city and can become a scene of a city. Now we still often build or rebuild multi-storey pagoda as a scenic sport. The restoration of the Leifeng Pagoda, one of the most famous ancient architectural structures in China, will be completed this year in Hangzhou, the capital of Zhejiang Province, in eastern China. And in Changshu, a big city in Jiangshu Province, It will build a new pagoda in Tianning Temple. This pagoda will be 150 meters high totally. Fortunately I take part in the architecture design of this pagoda.

Figures 8: Leifeng Pagoda (before it felled at 1924)

Figures 9: Simulation of the pagoda in Tianning Temple

Every multi-storey pagoda has his unique shape of his body. This shape show the character type of the pagoda, such as thin like young girl and fat as mature lady. Now we focus on the generation of the shape of a pagoda.

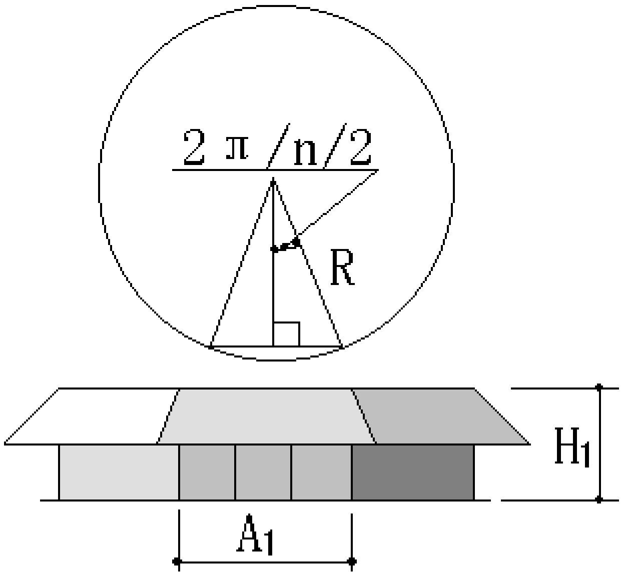
As mentioned earlier, multi-storey pagoda was just a serial of pavilion-like building added for effect and then more added until a tower-like shape was achieved. Each storey diminishing in which while simultaneously each order is reduced in height. So we just need decide the size of the pavilion on each level.

When we design a pagoda, the first precondition is the place where the pagoda will be located. It will decide the size of the end of the pagoda. As all the plane of pagodas is regular polygon, we can signify the size with the radius of the circumcircle, symbolic representation with **"R_1"**. After we decided the number of the sides **"n"**, we can get the length **"A_1"** of each side of the first floor of the pagoda.

$$\mathbf{A_1} = 2*\mathbf{R_1}*\mathrm{Sin}(2\pi/\mathbf{n}/2) = 2*\mathbf{R_1}*\mathrm{Sin}(\pi/\mathbf{n}) \qquad (1)$$

As each side is separated into three rooms (or bays, Jian in Chinese) by four columns, and each room is almost square on elevation. So the higher of the column of the first floor is third part of the length of the side. And the higher of the roof of this floor is equal the higher of the column. So the higher of the first floor **"H_1"** is:

$$\mathbf{H_1} = 1/3*\mathbf{A_1}*2 = 4*\mathbf{R_1}*\mathrm{Sin}(\pi/\mathbf{n})/3 \qquad (2)$$



Figures 10:The first floor of pagoda

Now we get the main size of the first floor: $\mathbf{R_1}$, $\mathbf{A_1}$ and $\mathbf{H_1}$. Then we decide the main size of the next floor: $\mathbf{R_2}$, $\mathbf{A_2}$ and $\mathbf{H_2}$. This time we begin at the higher of this floor. We can set a parameter **"K_1"** represent the reduce rate of the higher, so the higher of the second floor **"H_2"** is:

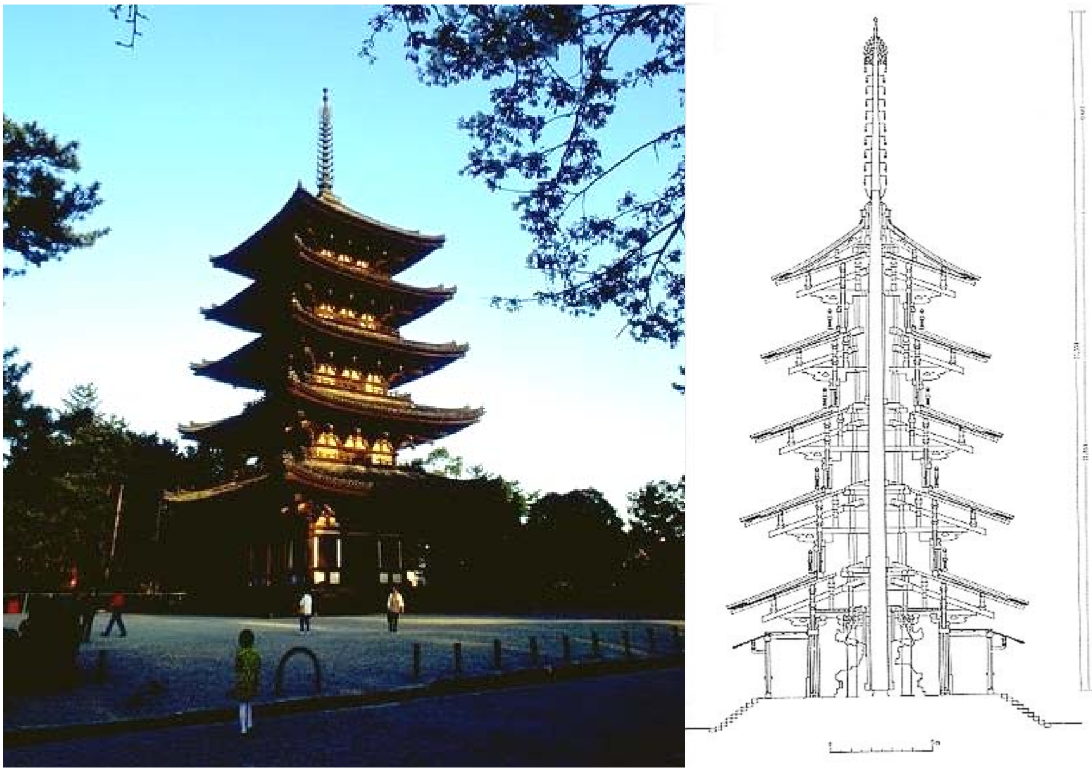
$$\mathbf{H_2} = \mathbf{H_1}*(100-\mathbf{K_1})\,\% = 4*\mathbf{R_1}*\mathrm{Sin}(\pi/\mathbf{n})/3*(100-\mathbf{K_1})\,\% \qquad (3)$$

Then, we can get other main size of the second floor:

$$\begin{aligned}\mathbf{A_2} &= 3*\mathbf{H_2}/2 = 1.5*\mathbf{H_2} = 1.5*(4*\mathbf{R_1}*\mathrm{Sin}(\pi/\mathbf{n})/3*(100-\mathbf{K_1})\,\%) \\ &= 2\mathbf{R_1}*\mathrm{Sin}(\pi/\mathbf{n})*(100-\mathbf{K_1})\,\% = \mathbf{A_1}*(100-\mathbf{K_1})\,\% \qquad (4)\end{aligned}$$

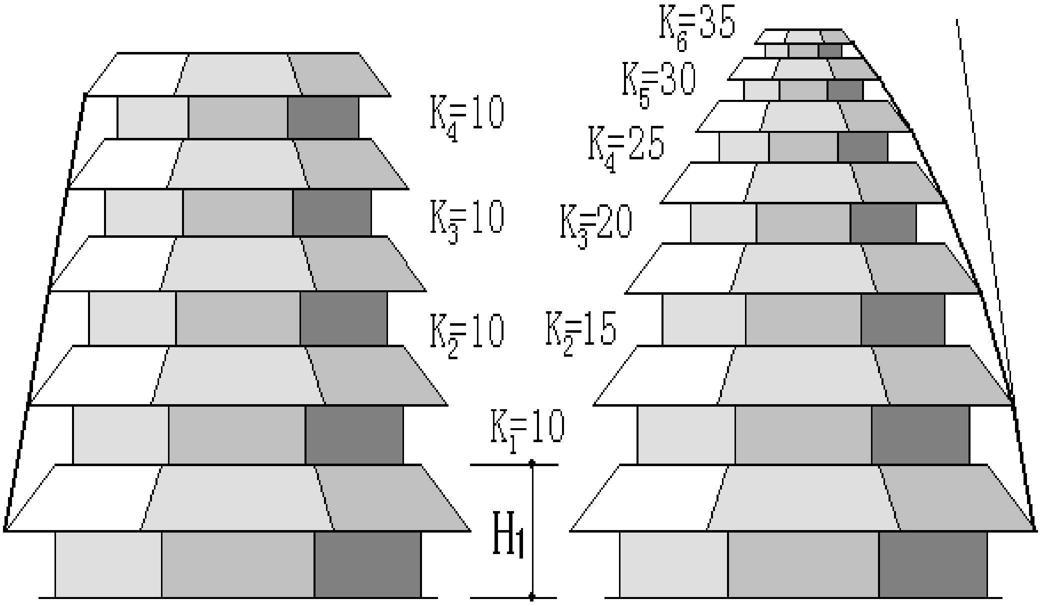
$$\begin{aligned}\mathbf{R_2} &= \mathbf{A_2}/(2*\mathrm{Sin}(\pi/\mathbf{n})) = 2\mathbf{R_1}*\mathrm{Sin}(\pi/\mathbf{n})*(100-\mathbf{K_1})\,\%/(2*\mathrm{Sin}(\pi/\mathbf{n})) \\ &= \mathbf{R_1}*(100-\mathbf{K_1})\,\% \qquad (5)\end{aligned}$$

Of course the reduce rates of the main size of the floor are equal, because they reduce both in plane and elevation.
Then we can set a parameter **“$\mathbf{K_2}$”** represent the reduce rate of the **3rd** floor, and **“$\mathbf{K_3}$”** for the **4th** floor, and **“$\mathbf{K_4}$”** for the **5th** floor, etc.
In the Tang Dynasty (618-906A.D.), pagodas were usually simple, square structures, and each floor has equal reduce rate, its’ mean “$\mathbf{K_1} = \mathbf{K_2} = \mathbf{K_3} = \mathbf{K_4} = \mathbf{K}$…”Though the original pagoda before the Tang Dynasty cannot be traced, we can still learn the shape of a pagoda in a temple during the early period of Buddhist development in China from Japanese temples. In Japan the pagodas were introduced from China with Buddhism. They are usually square in plan and five stories high, each story having its projecting roof. Generally made of wood, they exhibit superb carpentry craftsmanship. The Horyuji pagoda near Nara, of the 7th cent., is a noted example.



Figures11: The Horyuji pagoda near Nara, Japan

After the Tang Dynasty, the shape of the pagoda was developed. The reduce rates of the main size of each floor are no longer equal. They are increased continually, its’ mean “$\mathbf{K_1} < \mathbf{K_2} < \mathbf{K_3} < \mathbf{K_4} < \mathbf{K}$…”such as “$\mathbf{K_1}=10$, $\mathbf{K_2}=15$, $\mathbf{K_3}=20$, $\mathbf{K_4}= 25$…”So the top of the pagoda shrink more quickly. The outline of the shape becomes a parabola.



Figures12: Two pagodas with different parameter

Usually, the number series of the reduce rates are changed in accordance with the mystery traditional Chinese theory of numbers, such as “0, 9, 13, 17, 21…” The odd number means Yang in Chinese (Yang, male, sunward, positive, etc.).

Figures13: Tianning pagoda in Hebei

The actuality and the future

There are more than 3000 extant ancient pagodas in China now. Most of them are multi-storey pagodas. Each pagoda has his unique number series of the reduce rates. If we build the database of these pagodas, we can analyze these data and find the law of them. We can get the function of them, then use these control function we can generate the shape of pagodas automatically.

This method of the generation of the pagodas is very simple and rough. Further, we can set more parameter to control the shape, such as the ratio between the length of the side **"A"** and the higher of the floor **"H"**, the ratio between the higher of the column and roof, the ratio between the higher of the steeple and body, the ratio between the higher of the column and the depth of the eave, etc. Then we will make the generation of pagodas more accurately and freely.

Now we have no more chance to build a new traditional Chinese pagoda, but the spirit of these pagodas express the spirit of ancient China, it's also become a traditional Chinese spirit. The successful usage of the spirit is the shape of the 88-storey Jinmao Tower, located in Pudong (east of Huangpu River), in Shanghai.

Figures13: The Jinmao Tower

The 420.5-meter-tall building is the tallest building in China, the second tallest in Asia, houses the tallest hotel ever built and is currently the third tallest building in the world! It is considered combing the traditional Chinese architecture and world's latest technology.
Architects designed the building around the theme of the Chinese pagoda and the number eight, which the Chinese consider lucky. The lowest segment of the building is sixteen stories high and each succeeding segment is 1/8th smaller than its predecessor.
Except pagodas, the traditional Chinese architecture also has its inner law. If we study more of them with generate method, we will find more useful law to help us create new architect with traditional Chinese spirit. we hope more scholar all over the world spend more attention on both generate design and Chinese culture.