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Paper: **The Idea of Space, from Topology to Virtual Architecrue**

Topic: ***Mathematics, Architecture, Art***

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Abstract:

The discovery (or invention) of non-Euclidean geometry, topology and the higher dimensions (from the fourth on), the new idea of space to summarize, is one of the most interesting examples of the profound repercussions that mathematical ideas will have on culture, art and architecture. The paper will discuss the elements necessary to give sense to the word Space, starting from the new geometries and topology to Max Bill and the virtual architecture (Biennal of Venice of 2004, 2008 & 2010).

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The idea of Space, from Topology to Virtual Architecture

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1. Introduction

In the second half of the 19th century geometry had mutated significantly. In a letter of December 1799 Gauss wrote to Farkas Bolyai on his tentative to prove the Fifth Postulate of the Elements of Euclid: "My works are very advanced but the way in which I am moving is not conducing to the aim I am looking for, and that you say to have reached. It rather seems to put in doubt the exactness of geometry." Gauss never published his results on this particular topic in his lifetime. In 1827 he published the "Disquisitiones generales circa superficies curves" in which he introduce the idea of studying the geometry of a surface in a "local way" without minding its immersion in a three dimensional space, studying the invariant properties of the surfaces. Between 1830 and 1850 Lobacevskij and Bolyai built the first examples of non-Euclidean geometry, in which the famous fifth postulate by Euclid was not valid. For some years non-Euclidean geometry remained marginal to the field, a sort of unusual and curious form, until it was incorporated into and became an integral part of mathematics through the general ideas of G.F.B. Riemann (1826-1866). In 1854 Riemann held his famous dissertation entitled *Ueber die Hypothesen welche der Geometrie zur Grunde liegen* (On the hypotheses which lie at the foundation of geometry) before the faculty of the University of Göttingen (it was not published until 1867). In his presentation Riemann held a global vision of geometry as the study of varieties of any dimension in any kind of space. According to Riemann, geometry didn't necessarily need to deal with points or space in the traditional sense, but with sets of ordered n-ples.
In 1872 in his inauguration speech after becoming professor at Erlangen (known as the *Erlangen Program*), Felix Klein (1849-1925) described geometry as the study of the properties of figures with invariant character in respect to a particular group of

transformations. Consequently each classification of the groups of transformations became a codification of the different types of geometry. For example, Euclidean plane geometry is the study of the properties of the figures that remain invariant in respect to the group of rigid transformations of the plane, which is formed by translations and rotations.

2.SPACE IS MATHEMATICS

Jules Henri Poincaré held that “the geometrical axioms are neither synthetic a priori intuitions nor experimental facts. They are conventions. Our choice among all possible conventions is guided by experimental facts; but it remains free, and is only limited by the necessity of avoiding every contradiction, and thus it is that postulates may remain rigorously true even when the experimental laws, which have determined their adoption, are only approximate.
Poincaré, in *Analysis Situs* (Latin translation of the Greek), published in 1895, is also responsible for the official birth of the sector of mathematics that today is called *Topology*. Poincaré defined topology as the science that introduces us to the qualitative properties, of geometric figures not only in ordinary space, but also in more than 3-dimensional space. Adding the geometry of complex systems, fractal geometry, chaos theory and all of the “mathematical” images discovered (or invented) by mathematicians in the last thirty years using computer graphics, it is easy to see how mathematics has contributed to changing our concept of space - the space in which we live and the idea of space itself. Because mathematics is not merely a means of measurement in recipes, but has contributed, if not determined, the way in which we understand space on earth and in the universe.
It is interesting to note that the study of contemporary architecture begins with the instruments that mathematics and science make possible; more than technical instruments, cultural instruments. It is important to mention that the discovery (or invention) of non-Euclidean geometry and of the higher dimensions (from the fourth on), the new idea of space to summarize, is one of the most interesting examples of the profound repercussions that mathematical ideas will have on humanistic culture and on art. [1], [2], [3]

3. TOPOLOGY

Poincaré defined topology as the science that introduces us to the qualitative properties of the geometric figures not only in ordinary space but also in more than 3-dimensional space.Thus topology is the study of the properties of geometric figures that, undergoing intense distortions which cause them to lose all of their metric and projective qualities, for example shape and size, are still invariant. In other words, the geometrical figures maintain their qualitative properties. We can consider figures that are made of materials that can be arbitrarily deformed, that cannot be lacerated or welded: there are properties that these figures conserve even when they are deformed.
In 1858 the German mathematician and astronomer August Ferdinand Moebius described for the first time in a work presented to the Academy of Sciences in Paris a new surface of three-dimensional space, a surface that is known today as the *Moebius Strip*. In his work Moebius described how to build (quite simply) the surface that bears his name. Among other things, the Moebius Strip is the first example of a non-orientable surface – it is impossible to distinguish between two faces.

Artists and architects sensed some of the topological ideas in the past decades, first by artists, then much later by architects. These shapes, that so interested Max Bill in the 1930s [4], could not go unnoticed in architecture, although it took some time: until the diffusion of computer graphics, which allows the visualization of the mathematical objects discussed, thus giving concrete support to the intuition which otherwise, for the non-mathematician, is hard to grasp.

4. INTERNATIONAL VENICE ARCHITECTURE EXHIBITION 2004

The theme of the exhibition in Venice was *Metamorph*.
“Many of the great creative acts in art and science can be seen as fundamentally metamorphic, in the sense that they involve the conceptual re-shaping of ordering principles from one realm of human activity to another visual analogy. Seeing something as essentially similar to something else has served as a key tool in the fluid transformation of mental frameworks in every field of human endeavour. I used the expression `structural intuitions' to try to capture what I felt about the way in which such conceptual metamorphoses operate in the visual arts and the sciences. Is there anything that creators of artefacts and scientists share in their impulses, in their curiosity, in their desire to make communicative and functional images of what they see and strive to understand?
The expression `structural intuitions' attempts to capture what I tried to say in one phrase, namely that sculptors, architects, engineers, designers and scientists often share a deep involvement with the profound sense of involvement with the beguiling structures apparent in the configurations and processes of nature - both complex and simple. I think we gain a deep satisfaction from the perception of order within apparent chaos, a satisfaction that depends on the way that our brains have evolved mechanisms for the intuitive extraction of the underlying patterns, static and dynamic."
These are the words of Martin Kemp, an art historian specialized in the relationship between art and science, published in *Focus* ([5], one of the volumes that make up the catalogue of the 2004 Venice International Architecture Exhibition.
In his article Kemp writes mainly about architecture. The image accompanying Kemp's article is a project by Frank O. Gehry, an architect who obviously cannot be overlooked when discussing modern architecture, continuous transformation, unfinished architecture, and infinite architecture.
Kurt W. Forster, curator of the exhibit, discusses the great complexity, the enormous number of variations developed through essential technological innovations, the continuous surfaces in transformation. He cites Ian Stewart's article entitled *Nature's numbers: discovering order and Pattern in the Universe* (1995). Some key words: pattern, structure, motif, order, metamorphosis, variations, transformations, mathematics [6].
Forster observes on Gehry’s work: “What really interests Gehry is the process, in the sense of dynamic process used to achieve a structural and aesthetic result." These words, projects and ideas at the 2004 Exhibition were visually closely connected to the ties between mathematics, architecture, topology I am writing about. The layout of the pavilion of the Venice Exhibition was assigned to two famous architects: Hani Rashid and Lise Anne Couture. In an article for the catalogue entitled *Asymptote, the Architecture of Metamorph*, they summarized their project as follows: [7]
“Asymptote's transformation of the *Corderie* emerged from computer-generated morphing animation sequences derived from utilizing rules of perspective geometry with the actions and dynamics of torquing and `stringing' the space of the *Corderie*.”

One of the studies of the layout was described quite significantly as follows: “Study of the topological surface that develops in the space of the *Corderie* and determines the movements and the curvatures used in designing levels."
In 1992 the architect Eisenmann (who won the *Leone d'Oro* for his architectures at the 2004 exhibition) and his collaborators projected a skyscraper in Berlin, the *Max Reinhardt Haus*. The structure of the enormous building is based on the topological surface, the Moebius strip. In 1993 Ben van Berkel planned and built the *Moebius House*. So these two projects held the place of honour in the large hall of the *Corderie*, as if a reminder of an important step in contemporary architecture, in the idea of transformation, of metamorphosis. An explicit reference to topology.
Until a few years ago these were utopic projects - and many still are; architects enjoyed creating projects that were never carried out.

5. INTERNATIONAL VENICE ARCHITECTURE EXHIBITION 2008

In the section “Istallazioni” of the Biennale di Architettura di Venezia of 2008 van Berkel e UN Studio presented a large room “Il camerino di prova” again clearly inspired in the shape of the Moebius band. An enormous structure which filled up one big hall of the Arsenale.
Always at the Biennale of 2008 there was the project of Zaha Hadid and Patrick Schumacher, subdivided between a hall in the Corderie of the Arsenale in Venice and the halls of the Villa Malcontenta, one of Andrea Palladio’s most famous constructions, along the Brenta river, far away from Venice. Starting from the rigid geometric proportions of the Villa of Palladio, through a mathematical algorithm the authros realized forms in both the pre-existing buildings, the Corderie and the Villa, “a system of closed shapes whith which seduce and attract the closer world and even the most far away.” [8]
In november 2009 a new space for contemporary art and architecture was opened in Rome, the MAXXI. A project of the architect Zara Hadid who declared: [9] “First of all we had to decide if to mantain or not all the existing buildings. And after this decision we started to study the geometries which will substitute them, if they must be orthogonal, parallel or diagonal. The end result is a conjunction of lines of different geometries all present in the syte. So this is how it all began and a fluid interpreation of the space was the result.”
Fluidity is one key word in contemporary architecture. And without topology, the science of continuous transformations, these new forms would have beeen difficult to imagine.
Coming back to the Moebius band the new project for the National Library at Astana in Kazakistan is of great interest.
Bjarke Ingels, fondatore di BIG, atelier Danese che raccoglie decine di architetti, descrive con queste parole la struttura: [10] “The design of the National Library combines four universal archetypes across space and time into a new national symbol: the circle, the rotunda, the arch and the yurt are merged into the form of a Moebius strip. The clarity of the circle, the courtyard of the rotunda, the gateway of the arch and the soft silhouette of the yurt are combined to create a new national monument appearing local and universal, contemporary and timeless, unique and archetypal at the same time.”
The ideal addition to the perfect circle will be a series of public programs that simultaneously wraps the library on the outside as well as the inside, above as well as below. Twisting the public program into a continuous spiraling path tracing the library on all sides, creates an architectural organization that combines the virtues of

all 4 complimenting models. Like a Möbius strip, the public programs move seamlessly from the inside to the outside and from ground to the sky providing spectacular views of the surrounding landscape and growing city skyline.
The 2 interlocking structures: the perfect circle and the public spiral, create a building that transforms from a horizontal organization where library museum and support functions are placed next to each other, to a vertical organization where they are stacked on top of each other through a diagonal organization combining vertical hierarchy, horizontal connectivity and diagonal view lines. By wrapping the transforming composition of spaces with a continuous skin we create a Möbius strip volume where the facades move from inside to outside and back again.
"The envelope of The National Library transcends the traditional architectural categories such as wall and roof. Like a yurt the wall becomes the roof, which becomes floor, which becomes the wall again", said Thomas Christoffersen, the Project Leader .

6. TOWARD A VIRTUAL ARCHITECTURE

In the chapter *Topological Surfaces* Alicia Imperiale writes [11]: "The architects Ben van Berkel and Caroline Bos of UN Studio discuss the impact of new scientific discoveries on architecture. And the role of topology, from the architect's perspective: "Topology is the study of the behaviour of a structure of surfaces which undergo deformations. The surface registers the changes in the differential space-time leaps in a continuous deformation. This entails further potential for architectural deformation. Continuous deformation of a surface can lead to the intersection of external and internal planes in a continuous morphological mutation, exactly like in the Moebius Strip. Architects use this topological form to design houses, inserting differential fields of space and time into an otherwise static structure".
Naturally some words and ideas are changed in switching from a strictly scientific field to an artistic and architectonic one. But this is not a problem, nor a criticism. Ideas move freely and each person has the right to interpret and attempt, as with topology, to capture the essence.
Imperiale continues regarding the Moebius Strip:
"Van Berkel's house, inspired by the Moebius Strip (*Moebius House*), was designed as a programmatically continuous structure, that combines the continuous mutation of the dialectic sliding couples that flow into each other, from the interior to the exterior, from the activity of work to that of free time, from the fundamental to the non-fundamental structure."
During the same period Peiter Eisenman was designing the *Max Reinhardt Haus* in Berlin [12].
"The building, composed of arches, made up of intersecting and overlapping forms, presents a unified structure that separates, that compresses, transforms and finally comes back together on the horizontal plane at the height of the attic. The origin of the form is represented in the Moebius Strip. Just as the Moebius Strip folds two sides into one surface by folding on itself, the Max Reinhardt Haus denies the dialectic tradition between internal and external and confuses the distinction between public and private."[11]
In 2001 Di Cristina edited a book on *Architecture and Science* [13]. In her introduction *The Topological Tendency in Architecture* Di Cristina clarifies that
"The articles bear witness to the interweaving of this architectural neo-avant-garde with scientific mathematical thought, in particular topological thought: although no proper theory of topological architecture has yet been formulated, one could

nevertheless speak of a topological tendency in architects at both the theoretical and operative levels. What mainly interests architects theorizing the logic of curvability and pliability is the significance of the “event", of “evolution", of “process", or the innate dynamism in the fluid and flexible configurations of what is now called *topological architecture*.

7. CONCLUSIONS

Topological architecture means that *dynamic variation* of form, facilitated by information technology, by computer-assisted design, by animation software. The topologification of architectonic forms according to dynamic and complex configurations leads architectural design to a new and often spectacular plasticity, in the footsteps of the Baroque or organic Expressionism."

Stephen Perrella, one of the most interesting *virtual* architects, describes *Architectural Topology* as follows: [14]

“Architectural topology is the mutation of form, structure, context and programme into interwoven patterns and complex dynamics. Over the past several years, a design sensibility has unfolded whereby architectural surfaces and the topologising of form are being systematically explored and unfolded into various architectural programmes. Topological “space" differs from Cartesian space in that it imbricates temporal events-within form. Space then, is no longer a vacuum within which subjects and objects are contained, space is instead transformed into an interconnected, dense web of particularities and singularities better understood as substance or filled space.

This nexus also entails more specifically the pervasive deployment of teletechnology within praxis, leading to an usurping of the real (material) and an unintentional dependency on simulation." Observations in which ideas about geometry, topology, computer graphics, and space-time merge. Over the years the cultural nexus has been successful: new words, new meanings, new connections.

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