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Random Mathematical Approach in Architecture design Process.

Topic: Architecture

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[1] ^ Third Workshop on Monte Carlo Methods, Jun Liu, Professor of Statistics, Harvard University
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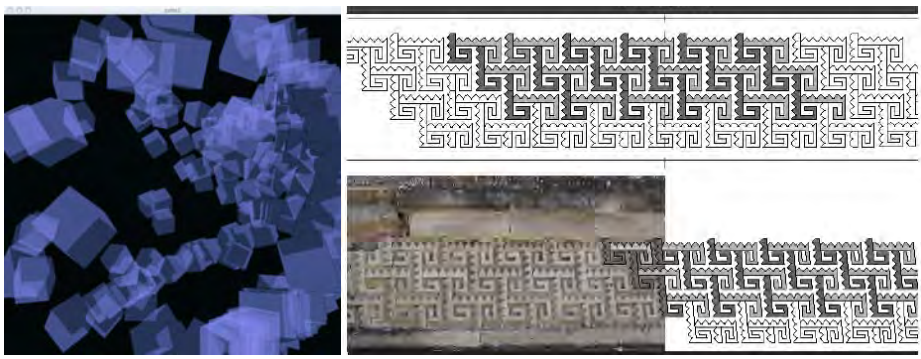
Abstract:
The actual issue about design process with link between Random Mathematics and design envelopes has a historical support in geometry on the frieze and panels design, express in mesoamerican architecture. The main idea is argued in mathematics and geometry with relation to arts and design of mesoamerican buildings. The mesoamerican civilizations like Mixteco-Zapoteco, Maya, Totonaca and others, are developed with methodology in their building process.. The mathematic term is geometry in symmetry operations for friezze and plans. The symmetry operation has a link with Randomness, however the symmetry operation has order and design pattern, instead of Random mathematics not or not as raises as the symmetry operation, but the fact is that the link is setting with infinite and finite models (for example like the image below shows).
Actually, we can see that the studies of Random Mathematics have many applications, especially on computer programs, for example Computer-aided design, the image below is a example of this. The block shapes are drawing by computer programming language randomly way and it's a finite model (but it could be infinite), the next image shows a development of pattern design on Mitla Facades with finite models, but with a chance to be infinite models.
There are many applications about Monte Carlo method on Design and visuals: "*…efficient in solving coupled integral differential equations of radiation fields and energy transport, and thus these methods have been used in global illumination computations that produce photo-realistic images of virtual 3D models, with applications in video games, architecture, design, computer generated films, and cinematic special effects*." (Szirmay-Kalos)
The hypothesis could be planted with this idea and others with Monte Carlo methods on Engineering and Design.



The firs image is produced by the processing programm, the next of is a design of facades in Mitla, Oaxaca.

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Keywords:
Randomness, Symmetry Operation, Finite Models, Infinite Models, Mitla, Oaxaca, randomly, Monte Carlo methods

Mathematical Approach In Architecture design Process A Historical View, from Mitla, Oaxaca

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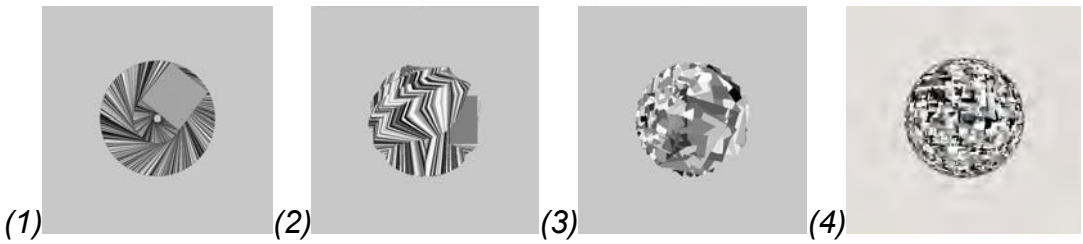


Figure 1.*Cubic transformations of images with a geometric dynamic behaviour, through programming language. The image (1) and (4) represent a simple process to draw shapes in 2D and 3D, created by computational language. Quadrangle to cube shapes has the same pattern to draw the 2D and 3D figure: circle and sphere.*

Abstract

Approach to mathematics, arts, computational programs and architecture was developed in the last century. A lot of research in Geometry, mathematics and Architecture show a complex geometries structures. The link between Mathematics and design was developed in Mexico. It has a historical background in geometry and design process in architecture, for example the frieze and panels design, especially in Mesoamerican architecture and specifically in Mitla, Oaxaca. The Mesoamerican design concepts like Mixteco, Zapoteco, Maya, Totonaca and others are developed with methodology on the symmetries conception. The mathematic term is symmetry operations for frieze and plans, and study is the discrete symmetries, whole numbers finite or infinite associated with the geometric pattern integrated in the design of a facade.

1. Introduction

The approach to the mathematical language to generate design in the facades is known by few. Consequently, little is studied about this term and about the symmetry operations in the architecture and the study of the discreet mathematics in the process of design, of a finite and infinite, organized and random way. The beginning of the mathematics with operations of discreet symmetries in design process has historical base in Mitla.

This study will be established in three parts. In the first one, I willexplain the historical base, in the second and third part I will focus on the current base in the study and on

the link of the geometry, the programs and exposition of experimentation across programs of design and generation of the forms.

The main idea originates from the study of an organized sequence through symmetry operations surrounding buildings with friezes and background in order to create random chaos pattern that can be observed in the series of images above (from 1 to 5).

"Symmetry is a general concept in mathematics; broadly speaking, a symmetry preserves a certain property (e.g., geometric similarity) of an object under some operation applied to the object. This notion of invariance is formalized in an elegant branch of mathematics called group theory. In the context of geometry, we will consider geometric transformations as the symmetry operations, such as reflections, translations, rotations, or combinations thereof."[1][1]

As a matter of fact, we can see that the studies of Randomness Mathematics have many applications, especially in computer-aided programs. The block shapes are drawnwith computer programming in a random way using a finite model (but it could be infinite as well). It is visible in development of pattern design on Mitla Facades with finite models, where infinite models may be used too.

There are many applications about Monte Carlo method on Design and visuals: "*… efficient in solving coupled integral differential equations of radiation fields and energy transport, and thus these methods have been used in global illumination computations that produce photo-realistic images of virtual 3D models, with applications in video games, architecture, design, computer generated films, and cinematic special effects*." [2]

The hypothesis could be planted with this idea using Monte Carlo methods on Engineering and Design.

2. A Historical View. Mitla'sSymmetry Architecture.

The esthetical development of the cultures in the ancient Mexico tackles their central subject matter in the study of the geometry and mathematics. Nowadays, it is observed and expressed in the facades of the buildings, especially in Mitla. The analysis derives from the link between the current knowledge of the geometry and the Mesoamerican architecture.

a) Mitla, Oaxaca

The city of Mitla has an area of 48 000 m². It is located in the town of San Pablo Villa of Mitla, in oriental limit of the central valley of Oaxaca (Vale of Tlacolula). Its geographical references are: 16 ° 55 ' of north latitude and 96 ° 24 ' longitude west. Literature shows that city of Mitla was inhabited in the year 500 B.C. (its first constructions date back to 200 A.D.) by the Zapoteca culture. It is quite probable that Mitla, in its initial stage,developed an exact geometry in composition and construction of its buildings, and in the course of the time it was perfected. At present, in the diverse friezes in its facades this knowledge is appreciated.

Apparently, 17 symmetry operations are studied for background, grouped by families, which can contain: reflections, slid reflections, rotations and, in all the cases, translations. Also, seven operations have been studied for friezes, they are not

designed to generate extensive planes neither to develop frames, and only they contain reflections, rotations and horizontal translations.

Mitla, has complex panels in its design, the image of the figure 2 shows the possible development of the design when tends to be infinite.

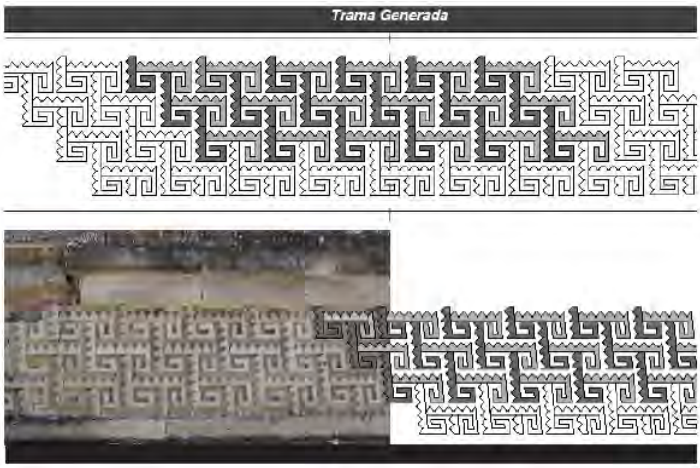


Figure 2. *Part of the east facade, Church group in Mitla. This image shows graphically a discreet symmetry, when the design of the motive in the panel tends to be infinite.*

The study of the friezes in Mitla is sustained in the idea that its architects were very advanced in process design, mathematics, geometry, and they had a highly developed visual perception.Moreover, they were applying beginning of proportion, harmony and symmetry in the configuration of its buildings and city. In the Mitla's architecture, the structures contain front's panels of friezes that show the beginning of a long way for this study: from construction system, design perception to scientific process.

b) Analysis Procedure

As a first approach to this topic was discussed two of the important features that presentedthe design of the friezes and the setting of thefacades: shafts of composition and the angles. The findings in the precision of the laying, disposition and job in each of its panels were surprising. In many of the cases the composition axes were coming closer to angles of thirty (30), forty five (45), sixty (60), ninety (90) and one hundred and eighty (180)degrees, and the disposition of these axes was generating geometric patterns like the square, rectangle, rhomb, triangle, parallelograms. On this matter, it is possible to state that the builders of this city handled geometric beginning or at least there was an attempt of approach, and not only that. They were probably applying such a complex way, generating movements in background and bands, where rotations existed as well as slides. Nevertheless, not in all the cases movement was happening in the background.In majority of cases, it can be noted that horizontal movements, rotations, reflections, but not vertical development with diagonalmovements.

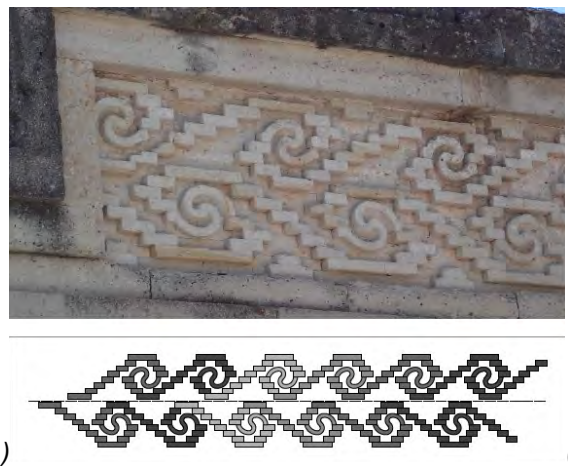
The friezes are built with parts of stone (in different cases) cut and modulated, willing, and summered in bands (see figure 3); however, there are carved friezes, located always in the lintels, waging a hollow.

For the design of panel there must have been some method of experimentation and prior stroke in site. The stroke on site could have been done by system of threads or natural fiber (as a system of wires). If the architects of Mitla based its delivery system on a pilot program before, they probably worked on a replica of the dashboard in stone, which enabled it to have a test run before the work and thus achieve greater accuracy in the final result.

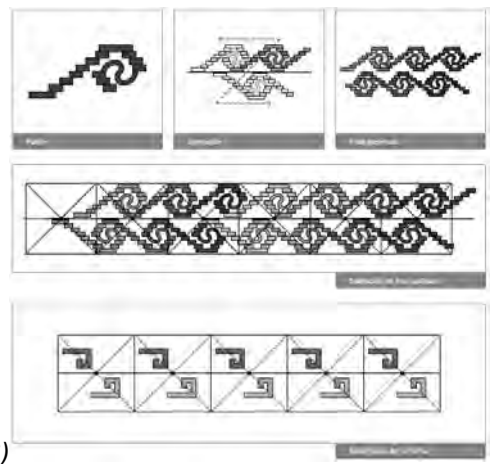
Figure 3.*The accommodation of parts is arranged in bands, cut into modules.*

If we take a pattern and apply the operation that corresponds with its transformation and then start with a basic design of element (is better if it is not a non symmetry design), we have the symmetry created by the transformations (Figure 4 A and B).

As a second step, we will take into account that all the patterns have a frieze of symmetry horizontal movement; once we create a basic unit that contains all the properties (other symmetries), we can move it in both directions. As a part of design process on facades, we take a basic pattern and reflect on a vertical line. It is recommended that you choose a close line but not the one that intersects your original item. And now there are two parts in your basic design: the original item and mirror image (see figure 5 (C) and (D).

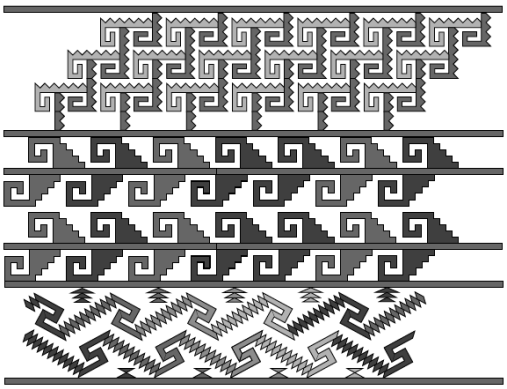


(A)



(B)

Figure 4. *In both images (A) and (B) one can observe the process of the classification of the marbles in one of the panels into buildings of Mitla. With the same procedure, each and every one of the panes in the architecture of Mitla, were classified. It should be noted that some of the special panels show differences in their design.*



(C)

(D)

Figure 5.*In images (C) and (D)the intention of the design is executed in background. It is an interesting process because it was a step closer to develop frames by plans.*

The evidence about 3D manipulated in Mitla's geometry is clear, however, we don't have specific evidence about 2D conception in every motive design, like plans or drawings.

The art and science transcended to other cultures of ancient Mexico as for example the Mayan culture, particularly Uxmal and Yaxchilán. These operations can be seen in the buildings, known as the quadrangle of the nuns in Uxmal (Yucatan) and the Palace of Yaxchilán (Chiapas).

3. Constructive Procedure

As mention above, the first approach to this topic was already discussed, two other important features that presented the design of the friezes and the setting of the facades are: shafts of composition and the angles.For example, in the picture below, it is shown that the parts are cut and embedded (approximately in the angles of 30, 60, 90 and 45 degrees). The frieze has axes of composition where they will be placing each of the parts; it is necessary to point out that the design of a same frieze has variations that are presented in another frieze of the same reason.

Taking up the theme of the approximation of the angles, some parts do not match in axles to accurate angles, these are at the top ready with a part number in 3 and 4(See Figure 7 E and F).

However, there is a geometric concordance between the parts. These are placed on distances and dimensions provided. This allows the implementation of the operation of symmetry in frieze and the formation of the entire design of motifs. Another observation that I would like to stress is that there is a transition between the band and frieze in the background, thus generating a scheme in frieze. Axis vary between lines 16, 17, 13 and 24, depending on the composition of frieze.

However, they are more frequentin the 16th line, where possibly there is a relation to the final design of the boarder.

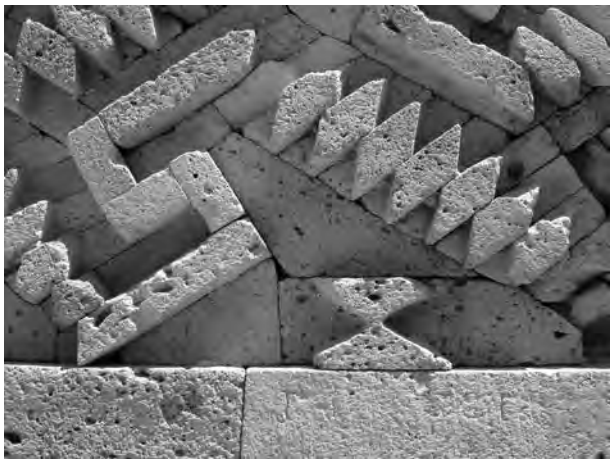
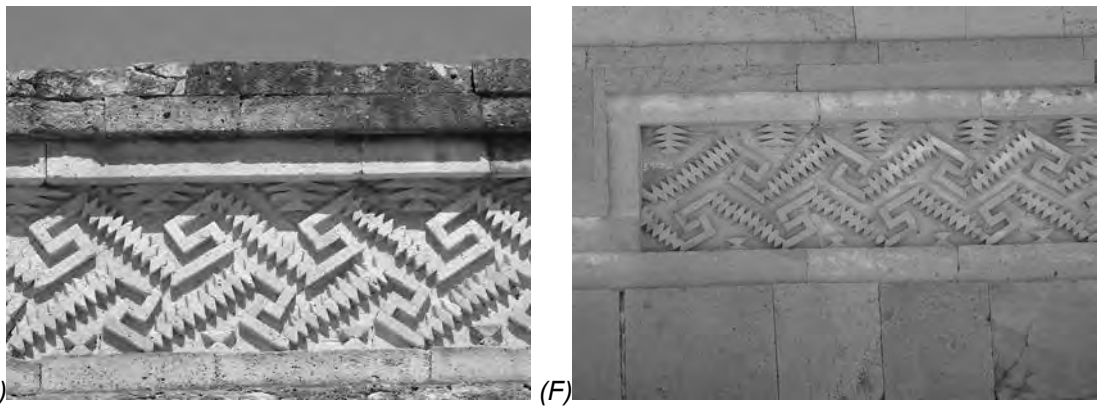


Figure 6. *We can see the disposition arrange little pieces of stone block with a specific cut with angles approximate to 90º, 30º*



(E) *(F)*

Figure 7.*The design of this panel with the motif of the "serpent" is another variation in the arrangement .The important detail is the number of elements. In the picture above, there are 4 triangles while in the photograph below we have only 3.correspond to the building of the columns and the group of the church respectively.*

The friezes not only show geometric principles of composition in band but they also portray a development in all background, a dual band and many of them show a transition between frieze and the background.

The foregoing description derives from the classification of the marbles in their designs for grounds, following an order based on groupings or on families. With this the study of the operations of symmetry in frieze it will be crucial to create a better interpretation of the geometric analysis of each one of the reasons prepared in facades .It is important to note that the classification of the marbles will be based on scientific research of the operations of symmetry, i.e. the majority of The fretworks present the operation of symmetry to friezes 'tg', some simultaneous dual-band , others to a single , and even are the principle of flat symmetry classification called '*pg*'.

To analyze the constructive system of structures it is necessary to know the type of material with which it is built. These structures were built with material taken from the region: stone quarry as can be seen in the photos

On the other hand, the task of pasteparts in some areas of the structures required the use of some kind of a primitive mortar.It was generally a mixture of mud with prickly pear(*'Baba de Nopal'* cactus) according to the oral tradition ; however, in site it can be seen in a few parts that mortar. For columns and other elements monolithic, the builders used the carved stone or chisel.

Some hypothesis suggest the use of the system to cover wooden base and a system of beams facts with a material similar to the species of a bamboo. At present, there is a room covered with this materialwhichproves this hypothesis (figure 8). However, the analysis of the space, reduced in size to the width, suggests that walls have been covered with the same material as well.

Figure 8. *Hypothesis of room cover suggests a permanent material versus imperishable material*

In most of the parts of the wall base they were trying to pressure the pieces of stone to configure the design the all wall; in many cases, the structures are available for an item that helped this accommodation to pressure from the parts, as can be seen in the photos above, in a few parts although there are parties are united with the kind of mortar manufacturing.

By observation and by the analysis on the site, you can infer that the structural system of the buildings was based on columns and decorated beams, what can only be seen in the interior of the tombs. It is possible that the foundation consisted of a system of rollers where heavy structure rested, a kind of procedure that developed a dynamic system in a seismic zone. The pillars were carved with jade stones or glass (according to oral tradition) that as discussed before, served as part of the foundations and also bore the cover of the rooms.

It is very likely that a system of drainage in the architectural ensemble of Mitla was developed, since in the buildings there is a drainage system organized by a few holes oriented toward the south-west, with the implication that by gravity and slope water returned to the stream.

This slope invested (term that designates the type of system) is a feature of the Yucatanhas buildings, especially in Uxmal. The profiles retract inward at the base of the wall, and are projected above the three friezes above, each more outgoing than the lower one. This seems to be an optical correction. In addition to resetting the force of the vertical lines in a block long and low, it allows the light reflected by the earth bathe the wall from the bottom; the result of this perception is that it produces an illusion of depth, in both the provision of the fretworks makes in a given time, the sun set to the work piece between figure and background (when you change the orientation of the lighting changes to Visual reading of the friezes on the facade).

4. Approach the Study on Mitla's Motif with QMCA[2]

As an important tool of analysis of the Mitla's motif we can use The Quasi-Monte Carlo method, with Stanford 3D Scanning, in recently years QMCA it have been used for scanners and surface reconstruction algorithms: *"Many researchers, however, do not have access to scanning facilities or dense polygonal models. The purpose of this repository is to make some range data and detailed reconstructions available to the public"*[3][3]

The researches of applications of QMCA are developed in arts and other 3D examples. It is suitable for open surfaces, and compares with others methods based on surface reconstruction.If analyze part by part the stripes in the symmetries of Mitla and as part of a basic pattern, breaks down, and then go piece-by-piece forming a complex system of design, we will have a possible application in the use of the QMCA.

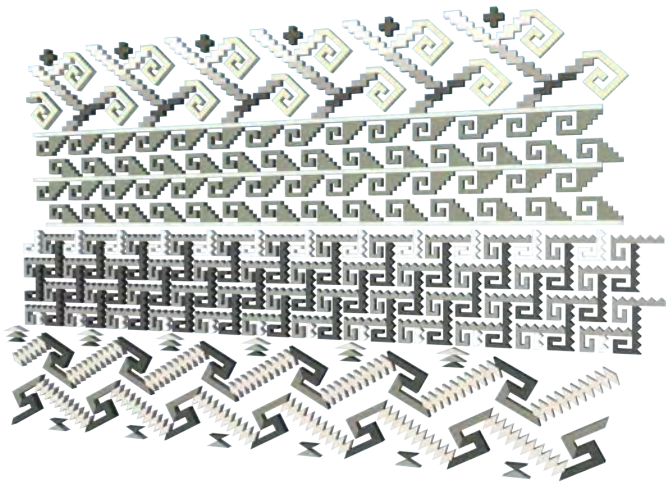
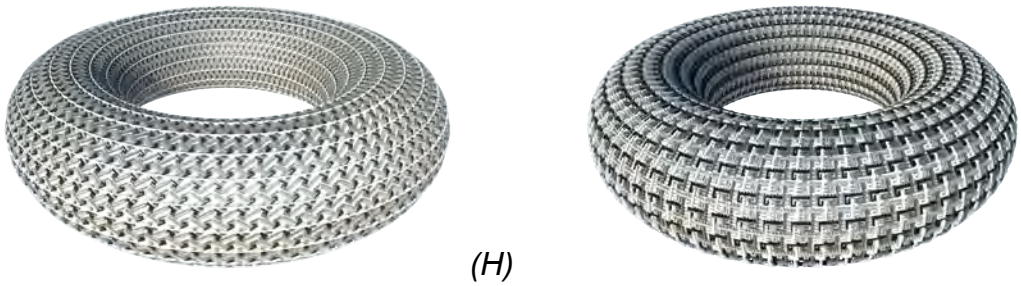


Figure 10. 3D Model about the complete background design composed with the friezes bands.

There are many methods to explore 3D spaces and the possibility to make transformations by spaces with symmetries, for example Mitra's studies about symmetry in design architecture, (see figure 9)show: *The surface of the model is sampled uniformly with average sample spacing h. The user parameter h determines the scale of the smallest symmetric elements that we want to detect. For every sample point we compute a local signature that compactly encodes local geometric properties at that point that are invariant under transformations of the specific*

transformation space under consideration. Sample points with similar signatures are paired and a canonical transformation that maps one sample to the other is computed and refined using local registration methods."… "*If a shape contains symmetries or repetitive structures, then the estimated transformations exhibit specific accumulation patterns when mapped to a suitable transformation space. These patterns can be extracted using cluster in methods and grid fitting techniques. While the method of [Mitra et al. 2006] is mostly concerned with pairwise symmetries, the structure discovery method of [Pauly et al. 2008] in addition analyses the spatial relations among different symmetries. The underlying formulation is based on theory of transformation groups and thus allows a rigorous mathematical treatment of the concept of structural regularity"* [4].

If we know about the procedure of their construction we can use it to design a model with the approximate measures in 3D and 2D to make transformations or, explore the possibilities in the management of the finite shapes, such as explore the symmetries of frieze into three-dimensional models such as the Torus, (see figure 9)



(G) *(H)*

Figure 9. *Overlapping the strips with design of the friezes at Mitla within the torus. The horizontal and vertical limits are closed.*

5. Conclusion

The study of geometry, mathematics and its possible applications through computer programs allow us to understand and progress toward the design process in architecture linked with the history of ancient civilizations and the ones that are already missing. The study of the operations of symmetry in the architecture of Mitla binds us the knowledge of the geometry with the architecture and provides the tools for designing facades, floors applicable to architectural projects. It provides mathematical principles that an architect should know for the design of objects with precise geometry. The mathematical principles bring us closer to explore ways by means of computer programs and invite us to reflect on the history of those ancient civilizations that possessed these processes of design without the use of technology.

The scientific breakthrough that these Mesoamerican civilizations achieved through its knowledge expressed in the design of its architecture by means of mathematics, geometry explored and used for its architecture.

It should be noted that the exploration still continues, since the scientific and artistic knowledge, extends to other Mesoamerican cultures. This analysis may provide the necessary tools for the design methods in the architecture (not only in the configuration of the walls and floors, but also in plant design architectural) driving a repertoire of possibilities in the symmetries of these, and also rescue this architecture.

6. Acknowledgements

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7. Notes

1 Symmetry in 3D Geometry: Extraction and Applications, Niloy J. Mitra, UCL

2 Professor Yu-Shen Liu call QMCA, that one method is based on the Monte Carlo integration and counts the number of intersection points between point sets and a set of straight lines. Russel E. Caflischsaid that “Monte Carlo is one of the most versatile and widely used numerical methods… Monte Carlo quadrature is attained using quasi-random
(also called low-discrepancy) sequences, which are a deterministic alternative to random or pseudo-random sequences. The points in a quasi-random sequence are correlated to provide greater uniformity”
3http://graphics.stanford.edu/data/3Dscanrep/. October 25 th 2013

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