Using the Inka's Calculator for Generative Art

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Abstract
The yupana is the Inca's abacus. It was first described by Juan Poma de Ayala in 1615 and recently by Radicati di Primeglio, Florio and Urton. So far, nobody is absolutely sure about how worked the yupana's computational system. In this paper I will present the theory of Peruvian researcher Davhit Prem, because it is the most efficient method to use the yupana with simple rules and movements similar to chess games or Turing Machines. I will suggest some ideas concerning the creative possibilities of the yupana for generative musical creation. The yupana's design and rules can be modified to represent musical information and to generate rhythms, melodies and structures of interesting complexity. All these possibilities add new formal solutions that expand what generative techniques such as Generative Grammars, Fractals or Cellular Automata usually provide. As a conclusion, I will add some insights about computation, generative art and cultural identity.

Introduction

The architecture of any technological artifact, such as analog or digital machines, has a deep impact over the generative process and the formal characteristics of its audiovisual outputs. Talking about the algorithms designed for music composition, Mozart wrote a process to compose random music using dice to choose harmonic and melodic rules[1]. Also the interaction with natural processes to foster imagination is an old and well-known practice with a deep impact over artistic production. I can mention here Leonardo's notes about the growing of humidity spots that can be used as inspiration to paint landscapes or skies [2]. The point is that every method produces results that reflect its information structure and the design of its architecture.

In this article I will try to explain the creative and generative possibilities of the yupana, the ancient Inca's calculator[2]. This can be of some importance not only for generative artists (always in search of new creative algorithms), but also for the definition of some concepts of generative art. But my goal is not just to produce new music; instead, I want to suggest new creative solutions that could develop creativity through cultural identity, a very important task for generative artists and educators.

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[1] "*The basis of the musical dice game consists of 272 musical measures and a table of rules used to select specific measures given a certain dice roll. The result is a randomly selected 16 bar minuet and 16 bar trio*". http://www.amaranthpublishing.com/MozartDiceGame.htm. Also Haydn developed something similar known as "philharmonic game".

[2] This continues the topics of my paper on Quipus that I presented at the XXI GA Conference (Roncoroni, 2018).

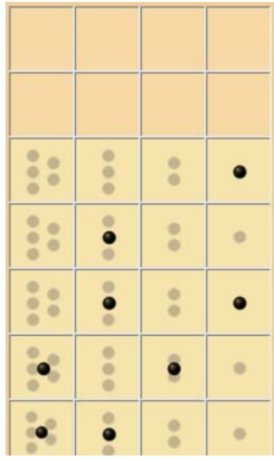
	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°	14°	15°	16°
2	96	22	141	41	105	122	11	30	70	121	26	9	112	49	109	14
3	32	6	128	63	146	46	134	81	117	39	126	56	174	18	116	83
4	69	95	158	13	153	55	110	24	66	139	15	132	73	58	145	79
5	40	17	113	85	161	2	159	100	90	176	7	34	67	160	52	170
6	148	74	163	45	80	97	36	107	25	143	64	125	76	136	1	93
7	104	157	27	167	154	68	118	91	138	71	150	29	101	162	23	151
8	152	60	171	53	99	133	21	127	16	155	57	175	43	168	89	172
9	119	84	114	50	140	86	169	94	120	88	48	166	51	115	72	111
10	98	142	42	156	75	129	62	123	65	77	19	82	137	38	149	8
11	3	87	165	61	135	47	147	33	102	4	31	164	144	59	173	78
12	54	130	10	103	28	37	106	5	35	20	108	92	12	124	44	131

Mozart: Musikalisches Würfelspiel (Berlín,1792). https://musescore.com.

I will start with a brief description of the yupana, following the system of Dhavit Prem [10], a Peruvian engineer that developed a fast and efficient method (in my opinion, the best) to use this artifact.
Then I will describe how the yupana could be used for generative art. As a reference for cultural identity, I choose the case of *icaros*, the Shipibo's[3] shamanic songs performed during ayawaska's sessions. The reason of this choice is because Shipibo's music is often based on the tetratonic scale that matches, as we will see, the yupana's design and topology [9].

The yupana

The yupana was first described by Juan Poma de Ayala in 1615. It is uncertain when it was invented; probably Incas inherited this technology from older empires conquered during their expansion.
Actually, there are many studies trying to explain how the yupana was used [4, 5, 7], but nobody can demonstrate which the right one is. I follow the theory of Prem because it is simple, efficient and powerful. Now, it could be safe to affirm that the simplest solution was the best choice for practical people like the Incas. In any case, it is the best method to implement the yupana computationally[4] that also opens a lot of generative possibilities.
The yupana is a matrix of 4 columns and n rows. The columns, from right to left, represent numbers 1, 2, 3, and 5 (note the Fibonacci's series). The rows represent, starting from the bottom line, units, tens, hundreds, thousands, and so on (theoretically, there are no upward limits).
To form a number, the *yupanki* (the yupana master) put little stones or seeds in the appropiate cells of the board, as shown below.



Quipucamayoc
https://www.arqueologiadelperu.com.
Number 13478

It's easy to see how a number, like 13478, is represented: with 1 seed in the first cell of row 10000 (the first row starting from the top), so we have 10000, 1 seed in the third cell of row 1000, so now we have 13000, then 1 seed in the first and third cells of the hundreds row, so we have now 13400, 1 seed in the second cell and 1 seed in the fifth cell of the tens row, so we have 13470, and finally, 1 seed in the fifth and another seed in the third column of the units row (the first from the bottom), so the number represented is 13478.

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[3] One of the most Important tribes of Peru's Amazon rainforest.

[4] If interested, send me an email to receive a copy of the Windows application.

With the yupana you can perform sums, subtractions, multiplications and divisions. I will explain Prem's method to show the procedures and functionality of the yupana. First some general rules:

- You can put any number of seeds in any cell of the matrix
- To compute, you must clear the yupana, that means to leave just 1 seed for cell and the fewer seeds that is possible
- This is done following simple rules that define how to move seeds in empty cells
- Every column has its own rules (the row's level doesn't matter)
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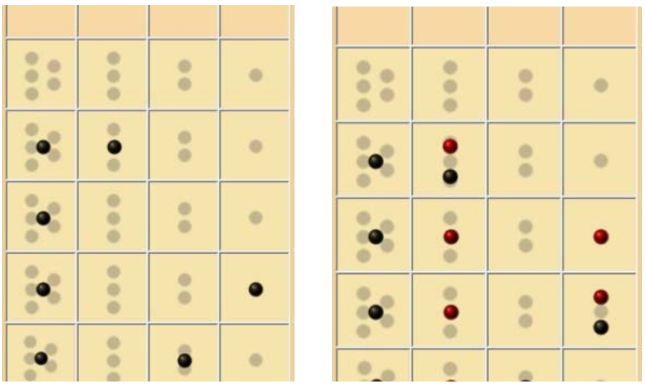


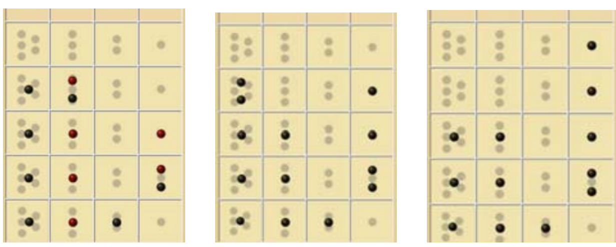
Number 8567
Number 8567 and 3443 (red)

It's not important here to expound all the rules, suffice to say that they are very simple and easy to remember. The yupana seems to be a concoct of chess games, Turing Machines and Generative Grammars[5].

To explain how to use the rules to obtain a correct computation I will perform the sum of 8567 and 3443. Remember: every cells of the yupana must contain only 1 seed per cell and the less number of seeds that is possible. When you accomplish this pattern, you have the result! Now, these are some of the rules, (enough to perform our sum):

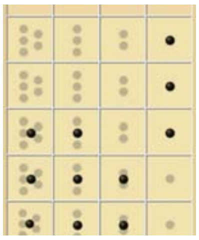
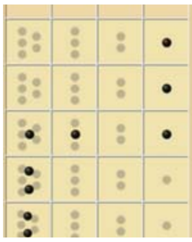
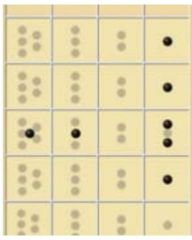
- Rule 1) Begin the process in any cell with 2 or more seeds
- Rule 2) With 2 seeds in the 3 column, you move 1 seed to the left and 1 to the right (actually the first column of the same row).
- Rule 3) With 2 seeds in the 1 column, you move 1 seed to the 2 column and discard the other; that makes sense, since 2 seeds in column 1 have the same value of 1 seed in column 2 (obviously in the same row!).
- Rule 4) If you have 2 seeds in the 5 column, you move 1 to the 1 column of the upper row and discard the other. 2 seeds in the fifth column (5x2) of row 1 have the same value of 1 seed in the first column of the upper row (1x10). By the way, this movement is called "pisqa".
- Rule 5) If you have a row of three cells with 1 or more seed for each, you do a jump: move the right seed two cells to the left, jumping 1 cell and discarding its seed. Like a piece of chess eating another.

So to make the computation we have the following sequence of moves (the color of the seeds is just to better visualize the numbers, it affects computation only in the subtraction's case):

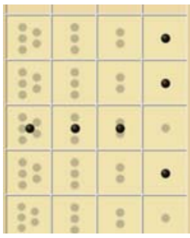
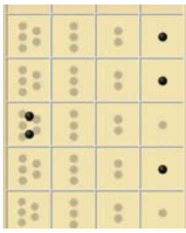
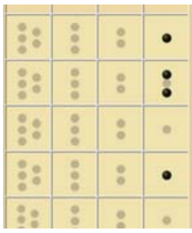


1. Rule 2 in row 4
2. Rule 4 in row 4
3. Rule 3 in row 2

[5] And also with Cellular Automata. CA concepts such as rules, values and neighborhood can be easily adapted for the yupana.

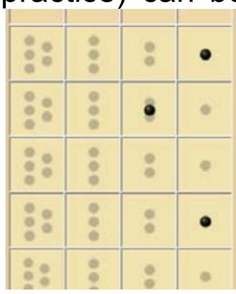




4. Rule 5 in 1-2 rows
5. Rule 4 in 1-2 rows
6. Rule 3 in row 3

7. Rule 5 in row 3
8. Rule 4 in row 3
9. Rule 3 in 4

With this last move we have the final result, as shown in the image below. We used a total of 9 moves, that (with some practice) can be done in a few seconds.



12010

The yupana and generative art

All this explains well enough some of the yupana's basics concepts. But why and how the yupana could be relevant for generative art?

In the first place, it is safe to say that generative artists are always seeking for new algorithms, data structures and production rules to improve their art. These elements are relevant for the creative capabilities that a generative process can provide and for the aesthetic characteristics of its musical or visual outputs. For instance, the data order in a Turing Machine's strip determines its computational behavior. In Cellular Automata, rules govern the evolution of its audio/visual pattern[6]. In fact, the rule's behavior generates in time a specific effect, even if you can't figure out the result from the beginning.

Turing (right) and colleagues working on the Ferranti Mark I Computer in 1951. This is how the first piece of computer music was created. https://www.bbc.com/news/magazine-37507707

On the other hand, taking into consideration that many (if not all) generative processes are recursive, it is correct to assume the existence of some sort of repetition. As Shannon demonstrated [14], repetition is a measure of the predictability of information, since it gives a rhythmic order that allows to forecast the information's flow. In other words, as Moles explained very well in *Aesthetics and Information Theory* [6], rules and data structures make the difference between noise and aesthetic order (at the cost of less information, since predictability and information capacity are inversely proportional). As Dobrian pointed out:

[6] A Turing Machine Music project: https://www.codechef.com/problems/TMBOX. Turing Machine Music: http://www.amancalledadam.com/?page_id=1753.

> "*In a totally random distribution of possibilities within a given range, all possibilities have an equal likelihood of occurring. In that case, over a large sampling they will tend to occur in equal amounts. This equal distribution is the musical equivalent of white noise. The random numbers can be shaped by control of their range, but the content of that range is neutral. To create a distribution in which some things are more likely to occur than others, we can ascribe different 'weightings' of probability to each possibility*" [3]

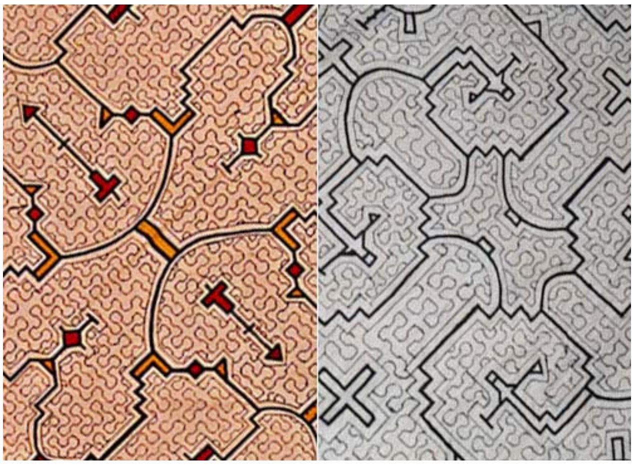
The yupana's pattern and rule set do just that: weightings control. The recursive application of rules, patterns and weightings spread an order that affects the form of the generative audiovisuals that the yupana produces. In fact, the yupana's cells' structure is like a Cellular Automata neighborhood: for any cell, we can check the data of every neighbors and act in consequence. For instance, you can dispose the cells in different ways (see the next paragraph), to represent different musical scales, processing flows or balances between randomness and determinism.
In any case, we can add complexity mimicking the strategies applied in Turing Machines, such as:

- Playing with different set of symbols (binary, number, letter, words…)
- Using various read/write rules for multiple tape heads
- Trying different initialing patterns
- Connecting tape heads with outside sources
- Connecting many Turing Machines in parallel

Another interesting property of the yupana is its design and computational logic. Both work like an interface to connect precise cultural contexts (the traditional and the digital), giving to the computational process cultural identity and originality. From the educational point of view, this makes the computational system significant to users of some specific cultural context, as suggested by Belaunde (translation is mine): "*It could be interesting to invert the roles and, instead to decipher kenes* (shipibo's textile drawings) *with occidental criteria, it will be more useful to analyze occidental writing with amazon eyes*"[1]. This concept could be generalized for ancient artifacts of every culture.

The yupana and Shipibo's *icaros*
To make a practical example of a generative production with the yupana, I will develop the case of Shipibo *icaros*. Shipibo is an important tribe of the Peruvian Amazon basin, and *icaros* are their shamanic songs or ayahuasca[7] ceremonies. Icaros are created using tetratonic or pentatonic scales with simple rhythm repetitions of dotted quaver and semiquaver sequences, mimicking the heartbeat pulse [7]. *Icaros* are linked with *kenes*:

[7] A hallucinogenic plant used by shamans to induce trance and improve consciousness.



Shamanes Shipibo. https://andina.pe
Diseños Shipibo https://www.dataisnature.com

> "*The intricate linear geometric and symmetrical artworks of the Shipibo Indians, a large tribe of the Peruvian Amazon, act as visual music maps – scores notating the chants and songs (Icaros) associated with Ayahuasca healing ceremonies.*
> *The textiles and embroidery, all crafted by women, contain recursive and self-reflective motifs, including geometric configurations common to those generated computationally by iterative functions*". [9]

Speaking about music, Belaunde adds (translation is mine):

> "*In some way it is possible to compare designs with musical scores. Nevertheless, this is a very generic comparison, because more than a writing form or music formally codified, the kene is a visual notation to aid memorization to inspire and remember and perform a song starting from some property or event included in a painted or embroidered, such as a curve or a straight line, or a line which change angle and direction*". [1]

But Shipibo's *kenes* and *icaros* are not just audio-visual patterns sharing recursive and fractal structures. In fact, both are more of a sort of shamanic languages to map land, rivers and trees, as Belaunde pointed out (translation is mine):
"*In the geographic scale, designs are linked to the rivers of the forest. From the botanic point of view, the designs represent the leaves, the veins and the midrib that transport the* savia *and the* rao *the plant's power. The veins of the leaf are actually named* ca*, radical of the word* cano*, path*". [1]

The computational function and the fractal pattern of *kenes* and *icaros* are, so far, evident. In the following paragraph I will experiment, using the properties of these cultural artifacts and the yupana, the possibility of a musical algorithmic solution.

The musical yupana: a hypothesis
The relationship between *icaros*, generative art and the yupana is a work of imagination and anthropologically

arbitrary[8], but I'm looking at its creative and generative potential. Thus, this is the starting point of a research on ethno mathematics and generative music that certainly demands more studies and experimentation [13]. As a starting point, I used the tetratonic scale because it fits the design of the yupana's board: the upper rows hold the higher frequencies and the lower rows the low frequencies. The order of the notes, from left to right, follows the tetratonic scale of A major [9].

A	C	D	E	A
huk	iskay	kimsa	piska	
D	E	G	A	D

D	E	G	A	Higher pitches
D Re	E Mi	G Sol	A La	A 440 MHz above n
D	E	G	A	Lower pitches

Anhemitonic (no half steps) tetratonic scale in A and C and positions in the yupana

In the second step, I used two yupanas, the first to determine pitches and the second to set durations, using only dotted quaver and semiquavers. Following Moles' musical information hierarchy, these are sonic objects, or the basic sonic elements, that put together, form the sonic cell [6].

Anyway, as in the traditional usage of the yupana, we need to design rules for every cell, considering that *icaros* have a limited frequency range, just 1 octave up or down, with a tendency to stay in the middle pitches. The musical yupana's rules behave in the same way of Turing Machines programs [16], context Generative Grammars[9] and it is also similar to the Markov Chains' mechanism that is widely used to make computer music [3]. If rules share a common logic when working with sonic objetcs and sonic cells, we can generate self-similarity and fractal patterns, like we can appreciate in *kenes*.

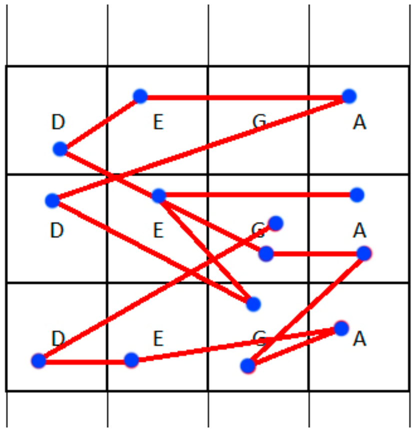


table 2 high, left, right, down
rule D: E or A down
rule E: D left or G right
rule G: E downor A right
rule A: G down or E left

table 1 middle left, right,
rule D: A up or E right
rule E: G down or A right
rule G: D left or A right
rule A: E left or G down

table 3 down, left, right, up
rule D: G up or E right
rule E: D left or G right
rule G: A right or D up
rule A: E left or D up

Turing Machine table. http://ocw.mit.edu. Rule pattern with musical yupana

Look at the similarity between the graphic pattern of *kenes* and Brownian Motion, but with an important difference: yupana's moves are not completely random, since rules generate an implicit order through repetition of information.

The yupana's generative potential

Concerning the generative domain, it is safe to say that a tool is really creative

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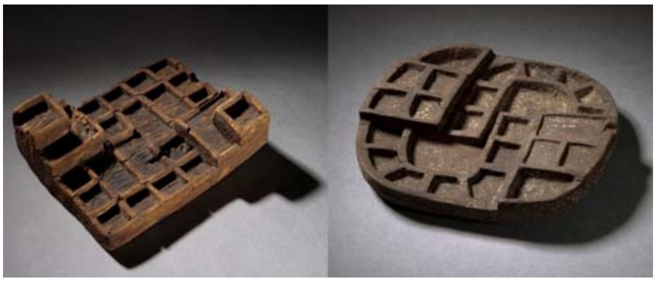
[8] The *icaros* and the yupana belong to completely different cultural contexts, even if Incas had contact with many Amazon tribes, Shipibo included.

[9] Context grammars are grammars which rules consider the neighborhood of the symbol (or cell in the case of the musical yupana). In this sense can be used like shape grammars and L-Systems.

when its architecture offers the widest range of possible combinations. This is the case of the yupana. I will mention here some of its possibilities.
Imagine to change the topology and use different designs to process sonic objects (pitches), bars (cells, or group of pitches) or higher level structures. Changing the geometry of the yupana obviously multiply its generative possibilities.
As you can see in the picture below, Incas made many yupana's designs, probably to serve not only as abacus, but as calendars or data bases. The yupana of the right shows a clear intention to control the flow of information and maybe to materialize an algorithm to process data. So far, each model could be used like a computer program to solve specific musical tasks with surprising results.



Two different yupana conserved in the Castello Sforzesco Museum of Milan, probably collected by A. Raimondi, a famous explorer of Peru born a century ago in Milan. http://elieducacion.blogspot.com.

On the second hand, we could use a set of parallel yupanas, adding a second one to process beats and a third to process measures. Like in Turing Machines, additional layers increase complexity and the creative potential.
And finally, the yupana physical model could be combined with natural elements (for instance, water). Clearly a similarity between the design of the yupana, Inca's architecture and landscape exists, as shown in the image below.

Tipón, Perú. Incas hydraulic facility and terraces. www.fotoaleph.com. It is unmistakable the similarity with the yupanas of the previous images.

Now, the flow of water is similar to the flow of information and can be combined with physical computing (sensors, actuators) and algorithms. As future computational yupana's applications, Prem pointed out (translation is mine): "*its specific ability to solve parallel computing problems allows the design of new algorithms and also of new architectures for math coprocessors*". [10]

Conclusions

The first goal of this paper is to discuss the yupana as a generative computational machine. I have explained how its features offer new possibilities to generate audiovisual artworks. The benefit of such an artifact is to insert the performer and

even the programmer in a different cultural dimension that opens new creative spaces, like African art did for Picasso and Braque. But the use of the yupana should not be limited to art production as such. We already have enough generative art made with Machine Learning, Artificial Life or Fractal systems. The addition of one more, even if exotic, is not the main priority. Besides, it seems to me that art in general is over saturated. Then, is more art (generative or not) really necessary?
What really interested me are some theoretical problems about art, computation and education that the study of the yupana clearly exposed. To mention one, to correct the generative art technocentric bias, in other words, to relocate its discourse in a broader dimension, connected to postmodern aesthetics and cultural production in general, seems to me more important.
Thus, the yupana's generative value is educational: the connection between different computational cultures and the inclusion of native knowledge that help artists today to face multiculturalism and globalization with better awareness. But how can we benefit native cultures without paternalism or neocolonialism [17]?
In fact, another important issue is to develop inclusive and sustainable education technology, a big problem for all native communities, especially in the Amazon basin. The digitalization of traditional computational devices, like quipus [8] and yupana, linking digital technology with cultural identity, makes the digital world significant for everybody.
Generative Art, from this point of view, is all about the design and development of intercultural, creative and interactive tools and procedures. In this sense, free Software and Open Source are politically strategic. But how students and teachers of native communities without refined digital literacy can read and understand algorithms and code?

My conclusion is that the generative art is wider than the domain constituted by algorithms and audiovisual objects: it includes code writing, interface design, semiotics and cultural studies. And when the users' creativity is empowered, the true meaning of the noun "generative" comes up.

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