Form Follows Algorithm: Differentiation of Chladni Patterns through Mathematical Functions in Processing

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Abstract

Derived from and inspired by Chladni patterns and cymatics, this study aims to contribute to the algorithmic visualizations of mathematical functions in computer programming environment which has led to exploration and creation of various patterns. Mathematical operations which derived from periodic functions associated with different variables to control patterns. Software called Cymatify is introduced which developed in Processing. Cymatify allows users to generate patterns by modifying mathematical functions and variables. In the scope of this study, outcomes of the pattern generation will be discussed in the context of complexity, unpredictability and diversity of the patterns.

Keywords: Chladni patterns, cymatics, periodic functions, pattern generation, Processing, graphical user interface (GUI), software.

Introduction

“If you want to find the secrets of the universe, think in terms of energy, frequency and vibration.” Nikola Tesla

The idea of creating patterns based on mathematical formulas emerged from examining the study of Chladni patterns. “First introduced by the German physicist Ernst Chladni in 1787, Chladni patterns refers to geometries resulted from different frequencies of sound” [1]. Chladni has investigated the behavior of sound waves and wave phenomena through an experiment setup where patterns emerge when solid particles scattered over a resonated plate in different frequencies [2] (*Figure 1*).

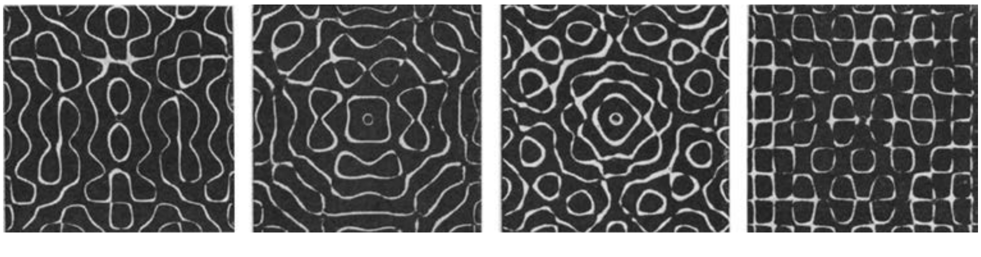


Figure 1. Chladni figures on square plates (Colwell, 1936, p. 4)

Emerged figures vary depending on the material type, size of the plate and the frequency of sound were used as variables in the experiment setup [3]. Chladni patterns are organic, complex, continuous and intricate figures which can be expressed with specific mathematical formulas. Studies conducted in this area are commonly based on the mathematical formula proposed by Robert Colwell [4] (1).

$$W = \left\{ \begin{array}{l} (\cos k_m a + \cosh k_m a)(\cos k_m x + \cosh k_m x) \\ + (\sin k_m a + \sinh k_m a)(\sin k_m x + \sinh k_m x) \end{array} \right\} \quad (1)$$

There are various digital simulations for visualizing Chladni patterns in digital environments. Main intension of these studies is to investigate and digitalize Chladni patterns as they emerge in physical environment. Therefore, they are similar to each other in terms of appearance, diversity and complexity. The purpose of the study is to propose a digital pattern generation framework based on the fundamentals of the Chladni patterns rather than mimicking or re-creating them. The wave phenomenon underlying reason of the emergence of Chladni patterns are investigated to this end.

Figure 2. Sound wave

Figure 3. Surface wave in water

Wave refers to the movement of a particle which follows a specific route [5]. There are various types of wave in the physical environment which we can exemplify with sound waves, string waves, radio waves, light waves, water waves etc. [6] (*Figure 2, 3*). In this study, wave type that is particularly emphasized is *sound wave* as it is in Chladni patterns. It is a must to define the wave phenomena with the mathematical expressions in digital environment to create the proposed framework. Sound waves are simply defined as sinusoidal plane waves which characterized by frequency, amplitude, speed of sound and direction. For this reason, waves can be described with the periodic functions. “In mathematics, a periodic function defines as a function that repeats its values in regular intervals or periods. The most common examples are the trigonometric functions, which repeat over intervals of 2*π* radians. Periodic functions are used throughout science to describe oscillations, waves, and other phenomena that exhibit periodicity” [7]. In this context, periodic functions are used as the fundamental mathematical basis in the study (*Figure 4, 5*).

Trigonometric functions	Non-smooth functions	Sinus-like functions	Vector-valued functions
Sine	Non-smooth functions	Trochoid	Vector-valued functions
Cosine	Triangle wave	Cycloid	Epitrochoid
Tangent	Sawtooth wave	Clausen function	Epicycloid
Cotangent	Square wave		Limaçon
Secant	Cycloid		Hypotrochoid
Cosecant	Tangent		Hypocycloid
Exsecant	Cotangent		Spirograph
Excosecant	Secant		
Versine	Cosecant		
Vercosine	Exsecant		
Coversine	Excosecant		

Figure 4. List of various periodic functions

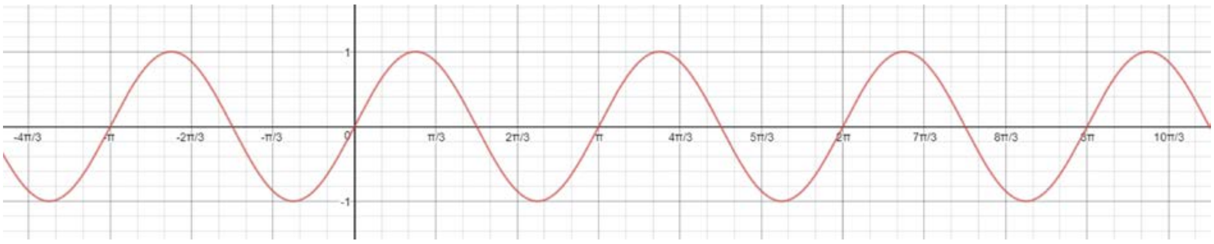


Figure 5. Sine wave is an example of periodic functions

The motivation of this study is to develop a pattern generation tool with an interactive, graphical user interface which users can generate and manipulate diverse patterns. Our hypotheses are, it is possible to generate patterns by visualizing periodic functions and infinite amount of patterns can be generated with Cymatify. In this manner, it is aimed to develop a software for generating patterns in the scope of proposed framework. We argue that the more we diversify the periodic functions the more various patterns can be generated. By using various periodic functions for figure generation, diverse and different patterns are expected to encounter than the original Chladni patterns. Outcomes will be discussed in the context of complexity, unpredictability and diversity of the patterns.

Related Works

Derived from Greek word '*kyma*', cymatics investigates wave phenomena and it is used to express the form of the responses of sound in diverse frequencies. In cymatics, Hans Jenny points out that "Typically the surface of a plate, diaphragm or membrane is vibrated, and regions of maximum and minimum displacement are made visible in a thin coating of particles, paste or liquid" [1]. Diverse patterns generated on the plate deriving from the geometry of the plate and the frequency values. In this sense, Chladni patterns can be classified as a part of a study of cymatics. In this part, cymatics and Chladni pattern studies will be examined.

Within a study about cymatics from Penn State University in 2014, a matrix and several control points are defined as a pattern generation system, and a morphology is presented with the effect of simulated waves. Generated matrix is deformed by various wave frequencies in the study conducted in the *Grasshopper* environment. Patterns are transformed into more complex figures by adding different wave functions into the algorithm [8]. Another approach which "Grasshopper" is used to visualize cymatics patterns is developed by Mohamed Dawod in 2014 [9]. Also, in 2014, a *Cymatic Display* medium is designed by Machine Histories to generate waveforms by the real time data from Audi's social media campaign [10]. This study is consisted from combination of physical and digital examination of cymatics. Patterns are generated in a physical experiment setup by the real time digital sound data.

Various studies are published by Albert Callejo between 2007 and 2008 to generate and animate Chladni patterns in *C++* and *Processing* environment [11]. In these studies where exciting results were obtained, the natural formation framework of Chladni patterns is followed by Callejo. In a study conducted by Enrique Zeleny in

2008, *Wolfram Alpha* computational knowledge engine is used to visualize Chladni patterns. This study aims to create software which allows people to manipulate and create several patterns. In this sense Zeleny's approach is considerably similar to ours. But in contrast to his work, our approach is to propose a new generation framework besides creating software. As he mentioned on his study, his approach is based on the Colwell's formula mentioned on the introduction part [12].

Similar to related works, we used the fundamentals of the study of cymatics. Apart from the examined studies, our goal is to investigate the differentiation possibilities of Chladni patterns. Moreover, another goal in our study is to design an interface which include users in this investigation process.

Cymatify: A Pattern Generator

In the Chladni's experiment, patterns are emerged as a result of relocation of the physical particles which are provoked by the vibrated metal plate by the sound waves. In contrast, in the digital environment, a direct connection is established between the sound waves and the digital particles. Main goal is to inform the digital particles by the sound waves coming from preset source points. This information includes the new location data, designated by the waves, for the points. Basic trigonometric periodic functions such as sine, cosine, and tangent are used to simulate sound waves in digital environment. Patterns are formed as a result of the displacement of those points *(Figure 6)*.

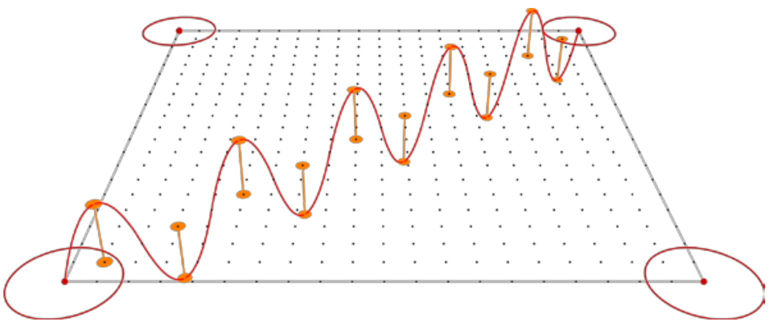


Figure 6. Matrix, source points, and a sine wave from a source

Processing 2.2.1 which is an open source programming software is used to digitalize the proposed pattern generation framework. Integrated graphical user interface allows users to create and manipulate various patterns. In this section methods and procedures will be explained, user interface will be introduced, and the outcomes will be discussed.

Method and Procedure

In order to achieve the pattern production, the proposed framework should be digitalized. For this purpose, the framework is defined as an algorithm in the Processing environment. The basis of the algorithm is consisted of a 2D matrix which each unit corresponds to a point on the coordinate system. Therefore, the number of columns and rows determines the *density* of the points. These points are the basic components that enable patterns to be produced. Generation occur as a result of the interaction of these points with various mathematical operations. In this context, it is possible to define these points as a *digital particles* which refer to the solid particles in the Chladni's experiment. The corner points of the matrix are defined as *source points*. Source points represent the origin points of the mathematical operations to be used in the pattern generation process. These mathematical operations define the *waves* which consisting of a combination of periodic functions and several variables. One of the variable is *distance* between the source points and the matrix points. In this context, *distance* cannot be directly manipulated due to its dependency on the location of the matrix points. By calculating the distance for each point, it is possible to determine the motion of the matrix points affected by the waves going out from the sources. The other variable is *frequency* value. Frequency defines the repetition of the periodic function in a unit of time. This variable can be directly manipulated in contrast to the d*istance* variable. Finally, the combination of the *frequency* value, the *distance*, and the *periodic function* forms the mathematical function (*Figure 7*). This mathematical function is applied on each point of the matrix. In the end, the operations determine the new locations of the points which resulted as emerge of the patterns *(Figure 8)*.

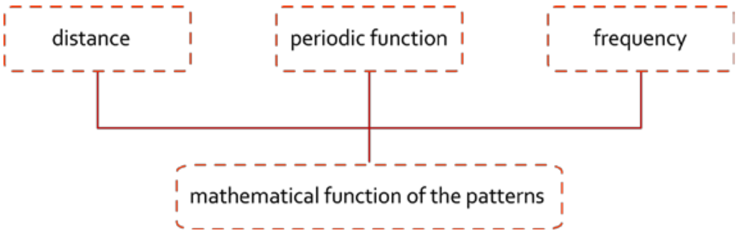


Figure 7. Creation the basis mathematical formula

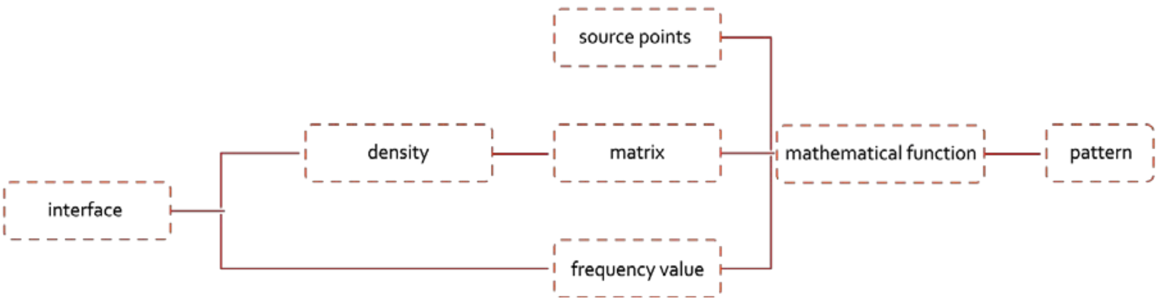


Figure 8. Pattern generation process

User Interface

Proposed interface of the software is consist of two parts. A window for displaying patterns and a menu for manipulating variables. In the menu part, there are 7 buttons placed to manage the type of the function (1), frequency value (2), size of the grid (3), the density of the grid (4), and color of the patterns (5) for generating and manipulating patterns. Users can choose and modify these variables with left-click or scroll through the buttons. Stroke button (6) can be used to stroke each grid in the pattern. Stroking patterns makes difference in low densities. Also, the interface allows users to export their generated patterns as PDF files. The program automatically creates a folder named as PDF, saves those files by automatically naming them as users click the export button (7), and assigns files a name according to date when patterns are created which include the year, month, day, hour and minute. (*Figure 9*).

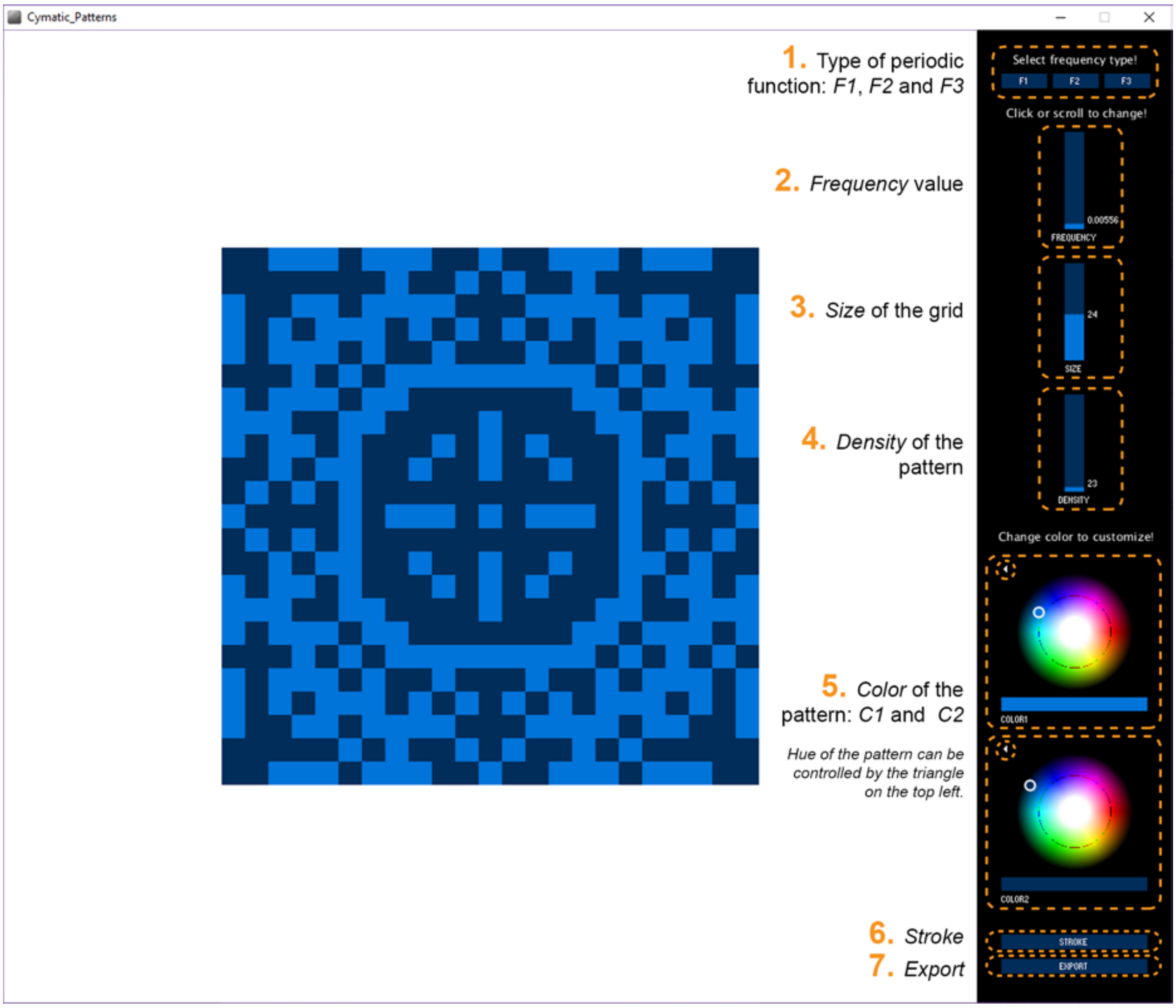


Figure 9. Introduction of the interface of Cymatify

Frequency type is used to change the type of periodic function in the mathematical expression. It allows users to make more distinctive manipulations on patterns. These periodic functions are predetermined in the algorithm. For this reason, the frequency type can only be manipulated by modifying the source code. Users are able to switch between 3 types of different mathematical functions which are expressed as *F1*, *F2*, and *F3*. *Frequency* slider controls the frequency value in the periodic function. *Size* slider gives users ability to change the size of the pattern. *Density* slider allows users to increase or decrease the number of rows and columns in the grid. As the density increases, the pattern complexity increases accordingly, and vice versa. *Color wheels* are responsible for the color of the patterns. *C1* and *C2* color wheels allow users to control the colors of the negative and positive areas separately. Cymatify allows users to create customized patterns by modifying the defined variables according to their intentions and function or place to be used.

Outcomes

In this section, various generated patterns through using Cymatify is presented (*Figure 10*). Generated patterns show that how making minor changes in variables can affect the diversity of the emerged figures. This variety among patterns reveals the importance of the precision of the variables. In the interface, to ensure precision 5 decimal places are used for generating intricate, different and infinite quantity of patterns.

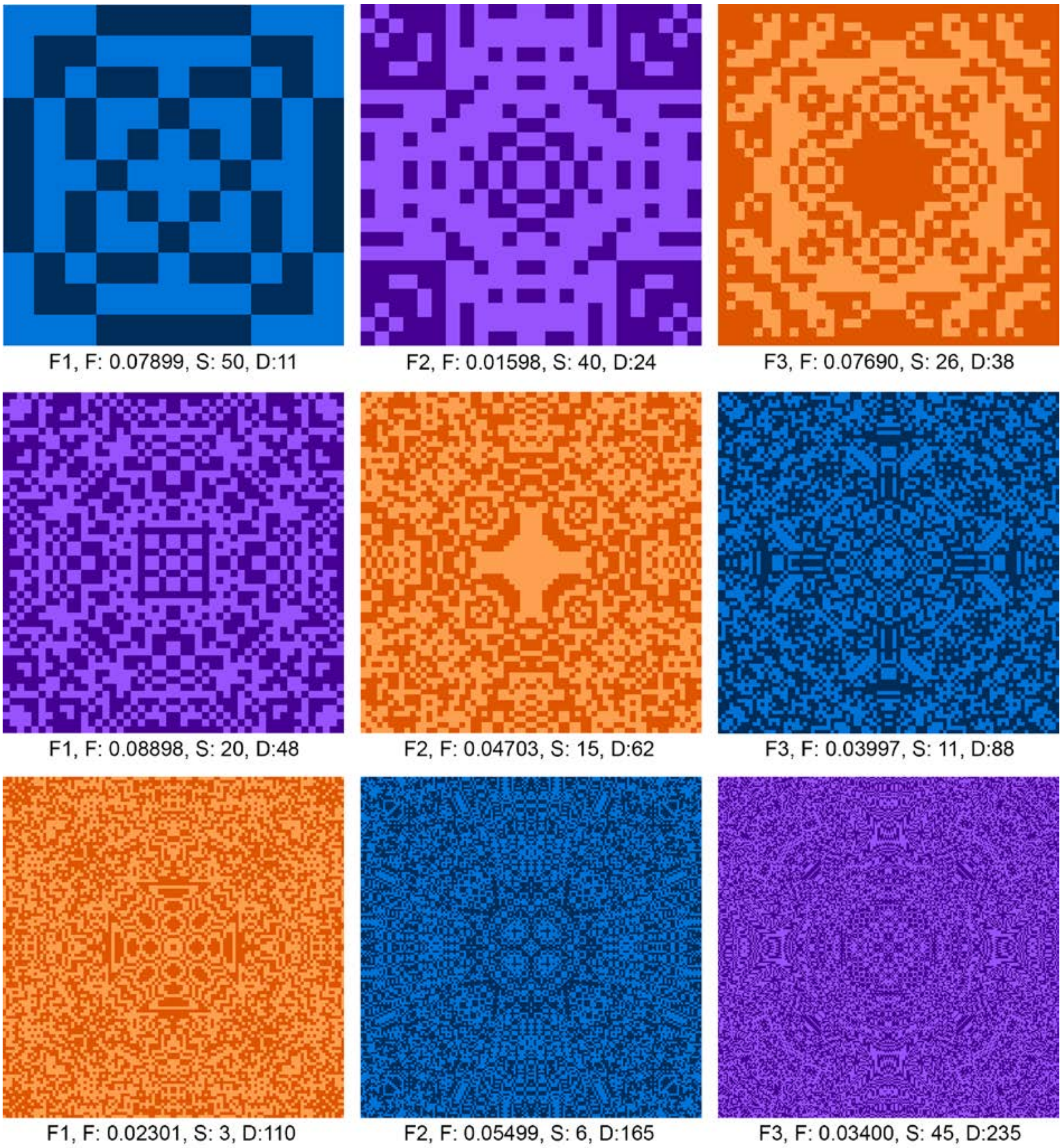


Figure 10. Outcomes are sorted in order to frequency and density values increasing from left to right. F1, F2 and F3: Frequency type, F: Frequency value, S: Size of the pattern, D: Density of the pattern

Results and Discussion

In this study, a novel pattern generation tool introduced with the Chladni's experiment as a source of inspiration. The underlying fundamental of the Chladni patterns is the wave phenomenon. It is possible to differentiate from Chladni patterns with the visualization of this phenomena within the proposed framework. Using periodic functions is a decent way to realize and visualize this phenomena. It is possible to create unique and intricate patterns in different complex structures. The resulting patterns can be varied by manipulating the *density* and *frequency* variables, and different combinations can be derived by the users. Furthermore, pattern variation can also be differentiated by diversifying the *periodic functions* in the source code of the software.

It is observed that the software is capable of producing infinite amount of diverse patterns hence the variety created within the periodic function. Changing frequency types, F1, F2 and F3, means changing the type of the periodic function which leads emergence of completely different patterns even though every variable is the same but frequency type (*Figure 11*). Moreover, infinite amount of patterns can be created by changing the *frequency* and *diversity* parameters since they have high precision for setting values. We can say that our two hypotheses are justified by the reasons mentioned above.

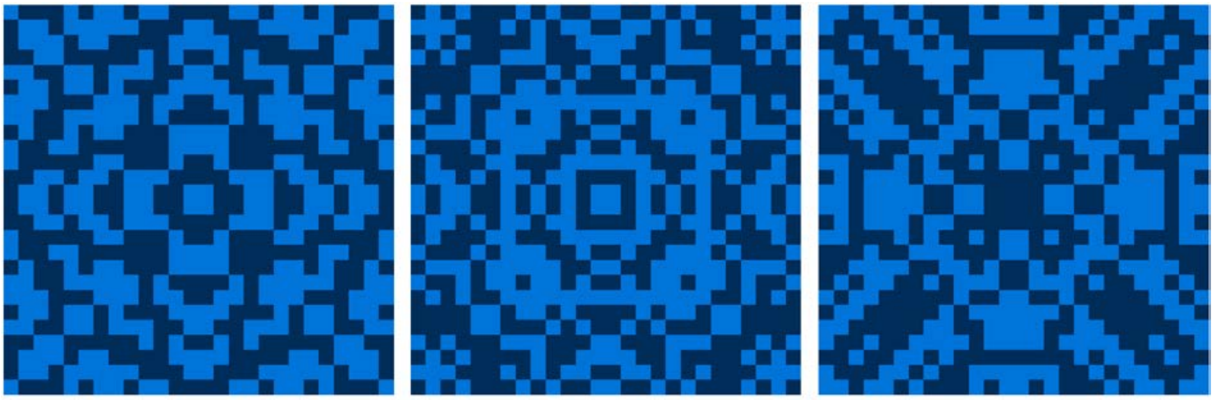
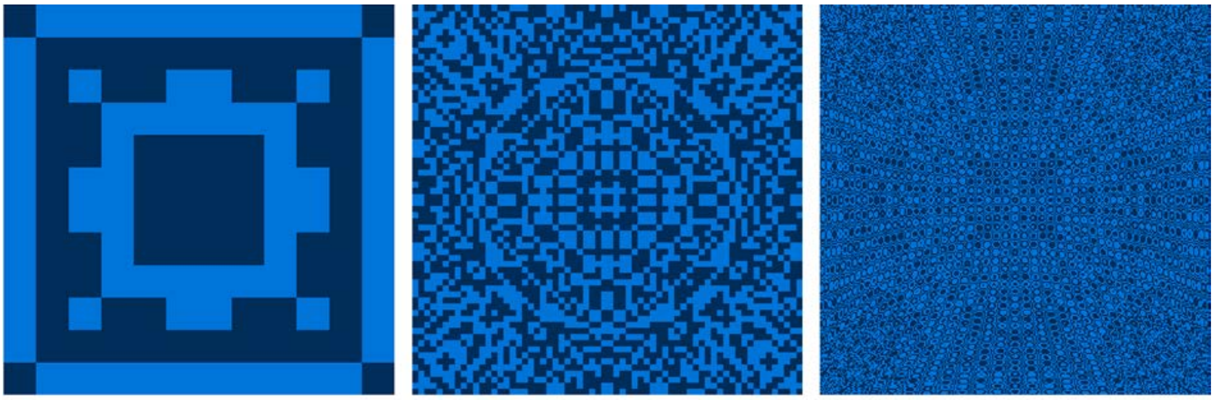


Figure 11. Values of the patterns variables are the same but frequency type

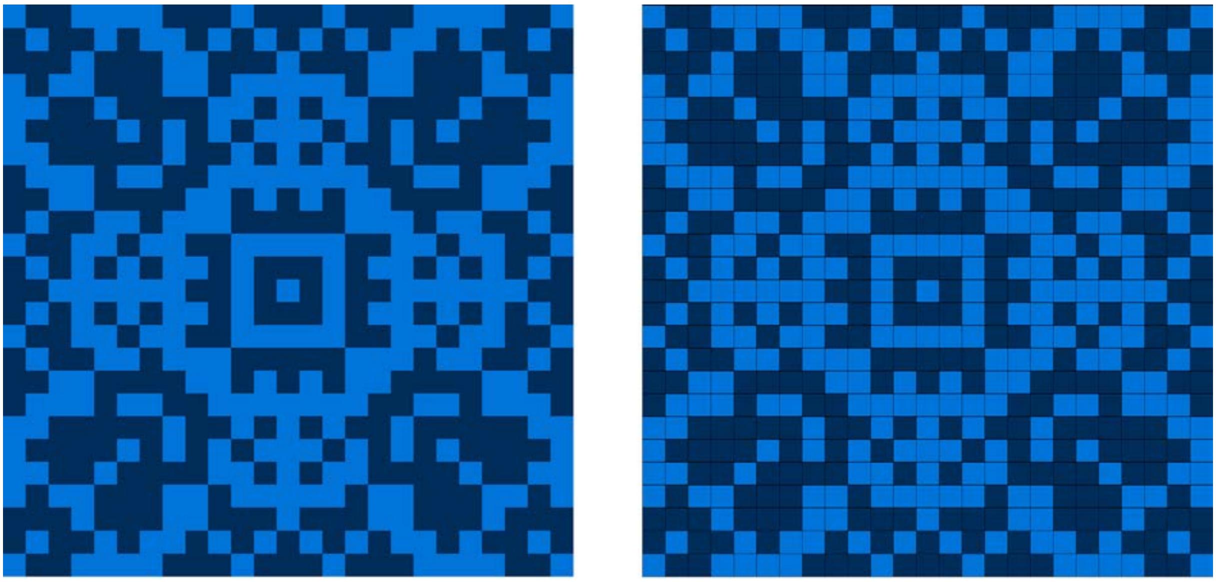
As a result it is possible to discuss outcomes of the pattern generation in the context of complexity, unpredictability, and diversity of the patterns. The variations within the created sequences can be provided by the variables on the interface. It is concluded that each manipulation on the provided variables are resulted as emerge of a pattern with a different appearance. Therefore, it is possible to argue that each variation on the functions and the variables has direct effect on the diversity of the patterns. Another point that is determined is, as the matrix size increases, which is represented as *density* in Cymatify, the complexity of the patterns increases (*Figure 12*). Each change in the mathematical function, frequency and density causes the generation of unpredictable patterns. In this context, it is correct to say that each variable is directly related to the unpredictability of the resultant product.



F1, F: 0.06500, S: 2, **D: 12** F1, F: 0.06500, S: 12, **D: 64** F1, F: 0.06500, S: 2, **D: 295**

Figure 12. Values of the patterns variables are the same but density

In the interface, by using stroke button black lines drawn around the each grid of the pattern (*Figure 13*).

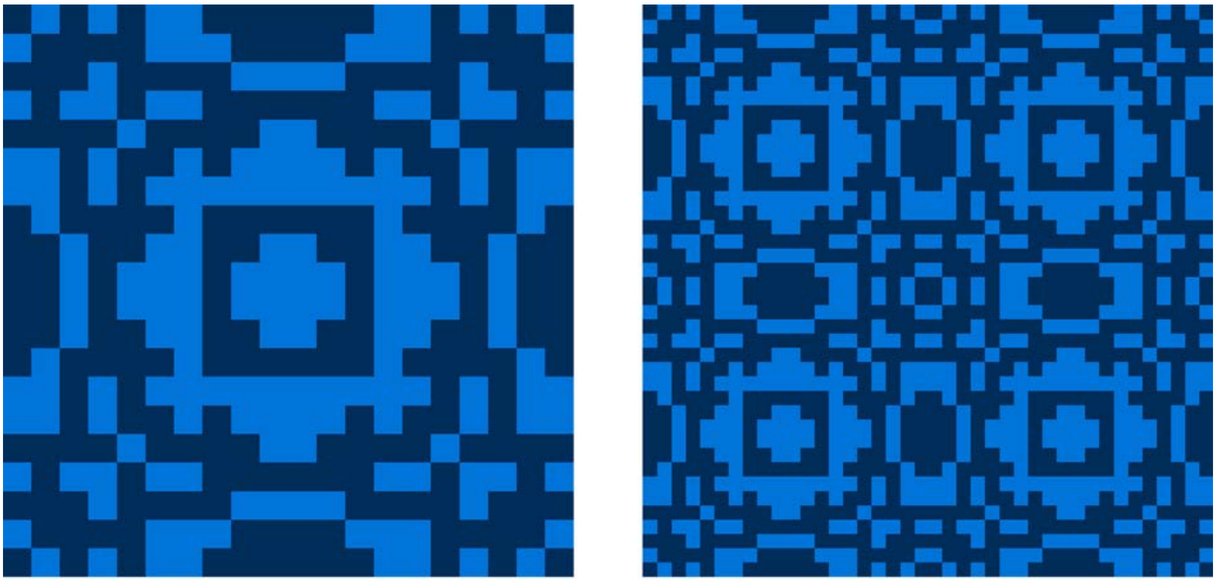


F1, F: 0.00719, S: 37, D: 25

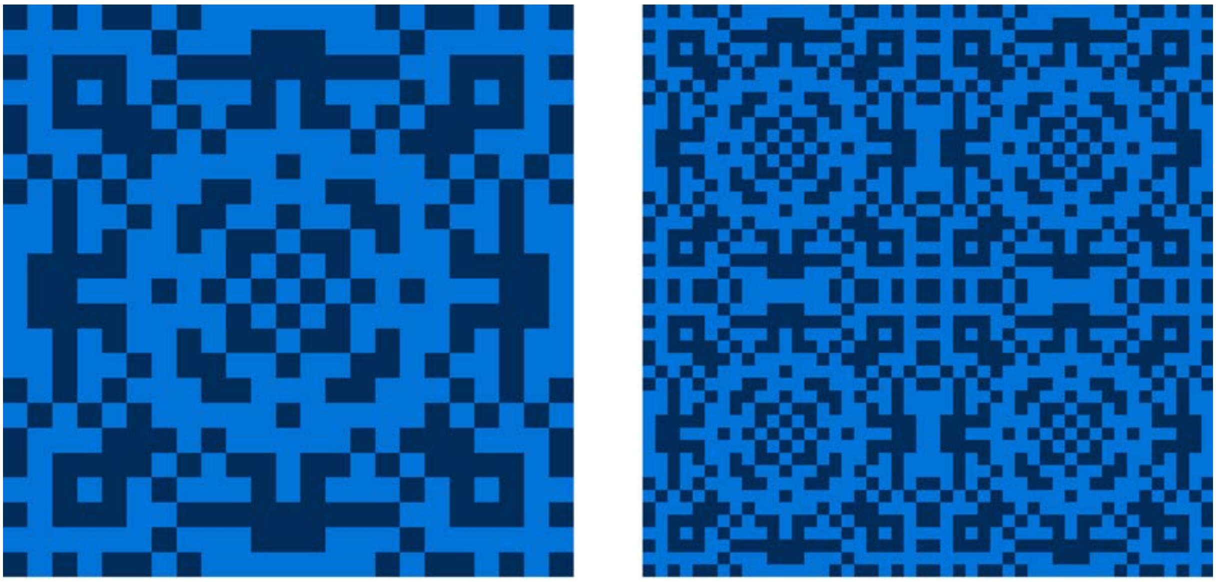
Figure 13. Difference between non-stroked and stroked patterns

Outcomes obtained from Cymatify are continuous patterns. Generated patterns showed continuity when brought side by side (*Figure 14*). Also, unnoticeable new patterns emerged in the intersection between the coupled patterns (*Figure 15*). This shows us variety can be increased by bunching patterns together.

Chladni patterns are usually produce under similar experimental conditions and digitalized with similar approaches. A few amount of diversity can be achieved by changing a small number of variables with limited values. For this reason, similar results are obtained as a result in the previous experiments and simulations. Thanks to the pattern generation framework introduced, it is possible to create complex, diverse and unpredictable patterns. Even if the proposed framework is based on a similar approach, due to the proposed differentiations in mathematical definitions, it is possible to achieve far more diverse and much more complex results. With the introduced software, unlike the Chladni patterns, it is possible to achieve an infinite number of patterns with the multiple combinations.



F1, F: 0.07000, S: 45, D: 20



F1, F: 0.03687, S: 45, D: 23

Figure 14. On the left, single pattern. On the right, pattern is copied 4 times

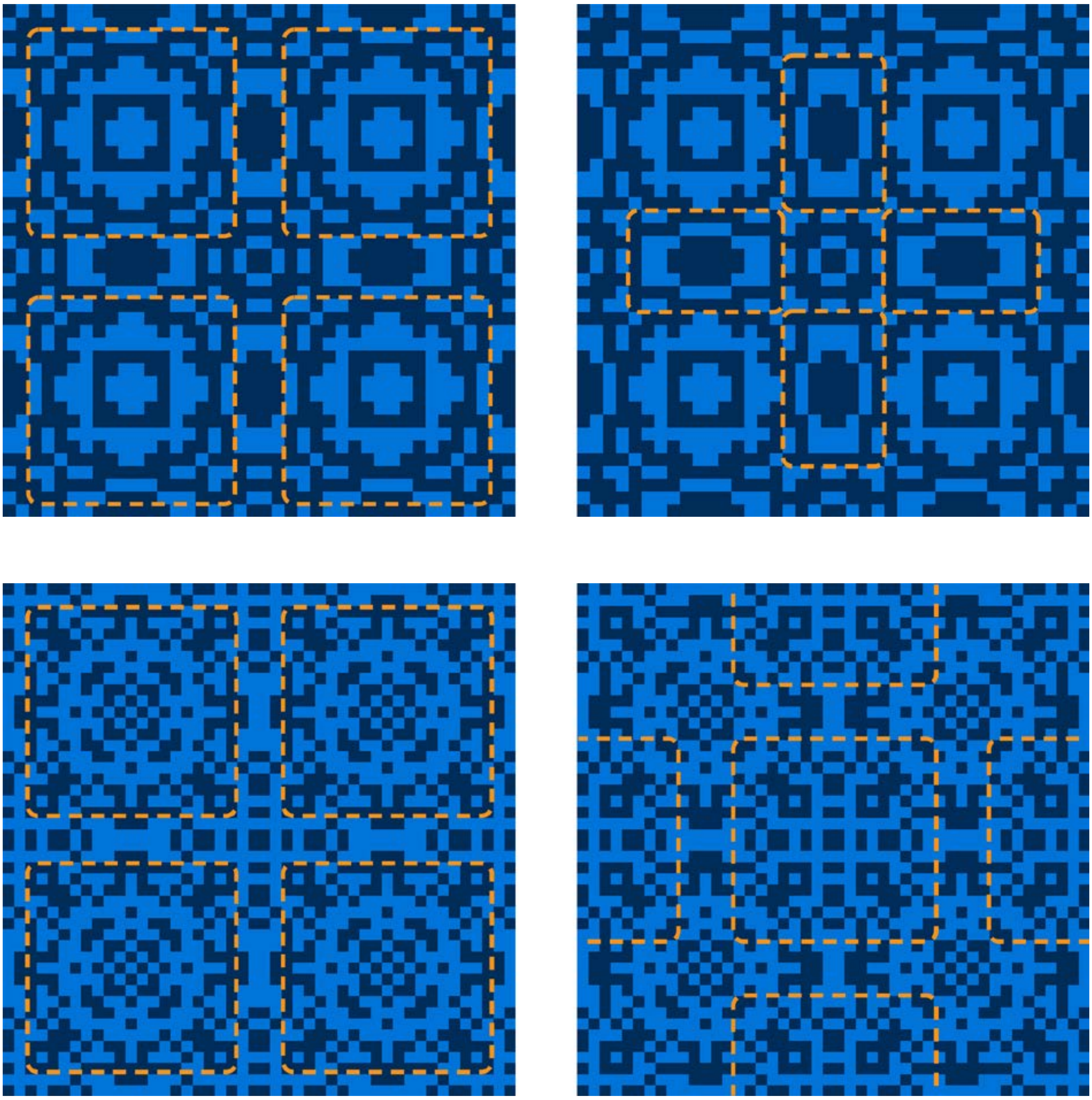


Figure 15. Existing patterns are on the left. Emerged new patterns are on the left

Future Works

It is determined that the one of the most dominant reason on the diversity of the patterns is the variations periodic functions. In this phase of the study, only a few *trigonometric periodic functions* is operated in the algorithm. In the future studies, it is aimed to increase the variety of periodic functions. It is predicted that different patterns can also be obtained by *Sinus-like, Non-smooth*, and *Vector-valued periodic functions*. It is planned to integrate more complex functions into the algorithm such as *Fourier series*.

This study includes two-dimensional pattern production in a 2D plane. As a next step, it is aimed to modify the algorithm in order to produce 3D patterns. In this context, we are carrying out various studies. By using the 3D modeling program *Rhinoceros* and the *Grasshopper* plug-in, proposed pattern generation method has been reconstructed in the 3D environment (*Figure 16*). It is predicted that various sectors such as architecture, product design, interior design and so on can benefit from the patterns that emerged as the result of this study. In addition, since these 3D patterns will be defined in the digital environment, it will also be possible to physically manufacture them easily by using devices like CNC and 3D printer (*Figure 17*).

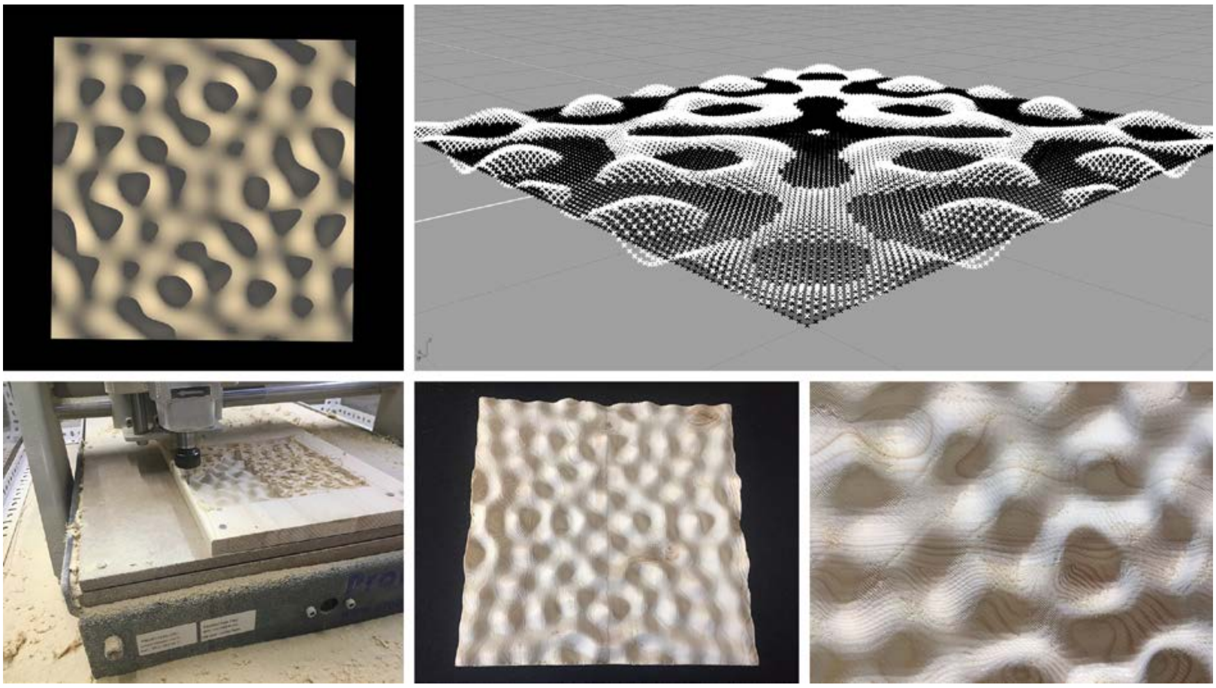


Figure 16, 17. Top: Digital 3D patterns. Bottom: Manufactured patterns by CNC

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