



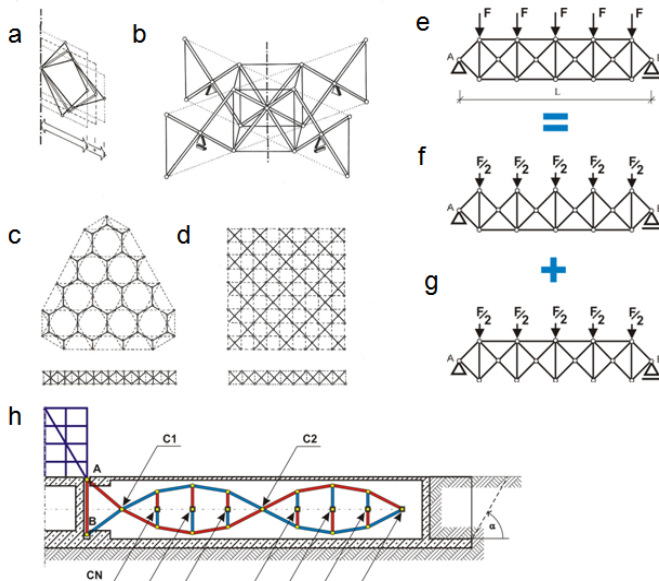
## EXAMPLES OF APPLICATION OF NUMERICAL MODELS IN ENGINEERING AND ARCHITECTONIC DESIGN

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### Abstract:

Spatial structures are widely applied in modern architecture and engineering since middle of XX century. They are defined as structures built by members quite uniformly arranged in space, while the forces between their component parts are transmitted also in spatial way. In spite of sometimes very complex forms, especially in case of tension-strut type of systems, they are mostly the very efficient structural systems of various types of buildings. Due to their application the buildings can get unique and interesting architectonic views. Numerical models defined by applications of various programming languages are very helpful in structural design of bearing structures [1]. Moreover application of principle of superposition can significantly enhance processes of design of various types of trusses [2]. Suitable application of this principle was basis during invention process of the two-stage method of calculation of statically indeterminate trusses as well as in the design process of structural concept of system of combined foundation. This type of foundation system makes possible to locate heavily loaded buildings on ground of small load carrying ability.



Images of a-d) selected type of tension-strut spatial structures, e-g) schemes of basic procedures of two-stage method of approximate calculation of statically indeterminate trusses, h) scheme of structural concept of the system of combined foundation

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**Key words:** Numerical model, programming language, space structure, design, architecture, engineering, roof, tall building

### Main References:

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## Examples of Application of Numerical Models in Engineering and Architectonic Design

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### Premise

Spatial structures are widely applied in modern architecture and engineering since middle of XX century. They are defined as structures built by members quite uniformly arranged in space, while the forces between their component parts are transmitted also in spatial way. In spite of sometimes very complex forms, especially in case of tension-strut type of systems, they are mostly the very efficient structural systems of various types of buildings. Due to their application the buildings can get unique and interesting architectonic views. Numerical models defined by applications of various programming languages are very helpful in structural design of bearing structures. Moreover application of principle of superposition can significantly enhance processes of design of various types of trusses. Suitable application of this principle was basis during invention process of the two-stage method of calculation of statically indeterminate trusses as well as in the design process of structural concept of system of combined foundation. This type of foundation system makes possible to locate heavily loaded buildings on ground of small load carrying ability.

The paper presents selected examples of defining numerical models of statically indeterminate trusses, which then are subjected to suitable processes of calculation of forces acting in their members. Moreover there are presented proposals of shaping of structural systems of multi-storey buildings including the innovative system of the combined foundation. An example of program defined in programming language Fortran the numerical model of selected type of structural system is also presented.

## 1. Concept of two stage method of calculation of statically indeterminate trusses

The proposed two-stage method of calculation of the statically indeterminate trusses was invented by the author during the preliminary analysis of a group of the spatial tension-strut structures, schemes of which are shown in Fig. 1 a-d. These structures are built by means of struts, which constitute all their cross-braces, and by means of tension members, which are located in outer layers and moreover being their vertical members. Structural system built in this way has to be suitably pre-stressed. Simplified scheme of vertical cross-section of a basic truss system, representing this group of structural systems, is shown in Fig. 1e. It represents a plane truss being a four-fold statically indeterminate structure.

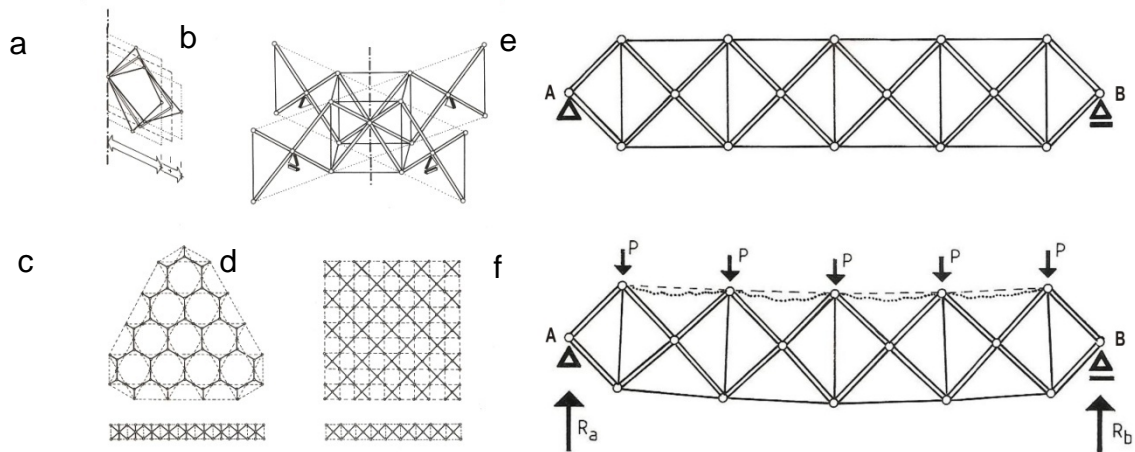


Fig. 1. a-d) Selected type of tension-strut spatial structures, e) scheme of cross-section, f) shape of bending of overloaded tension-strut structure

From analysis of Fig. 1f follows that certain number of the upper chord members are not able to take the compression forces, because they are constructed as cables, what implies that they are excluded from process of the force transmission. This type of structural system has to be calculated by application of sophisticated and complex methods. Number of the excluded members equals the degree of statically indeterminacy of the basic truss, see Fig. 1e. It means that the basic truss is the four-fold statically indeterminate system. From the general, basic conditions of equilibrium follows, that the approximate calculations of forces acting in particular members of such truss could be carried out in two suitable stages, see Fig. 2, by application of principle of superposition. The point of this method is that static calculations are carried out in two independent stages for statically determinate trusses, shapes of which are received through remove from space of the basic truss the number of members equal to statically indeterminacy of this truss.

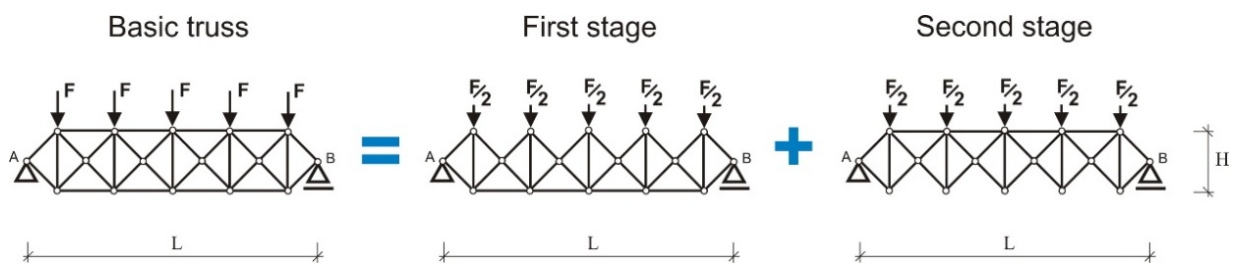


Fig. 2. Schemes of two stages of the proposed method of calculation

It implies that in the first and second stage there are considered the statically determinate trusses, what further implies that in both stages can be used one of very simple method of the force calculations, for instance the Cremona's method. In order to verify correctness of the two-stage method there were carried out some computations of simple form of the plane statically indeterminate truss having the same shape like the basic truss shown in Fig. 1e and in Fig. 2. It has the clear span equals 5.00 meters and the construction depth equal to 1.00 meter. In the basic case the truss is loaded in symmetrically way by means of concentrated forces applied to all nodes of the upper chord, each of value 1.00 kN. In the first stage four members of the upper chord are removed and concentrated forces of value equal to 0.50 kN are applied to all nodes of the upper chord, see Fig. 3. In the second stage, like previously, four members are rejected but this time from the lower chord of the basic truss and the statically determinate form of truss is loaded by concentrated

forces, each of value equal to 0.50 kN and applied to each node of the upper layer, see Fig. 4 The own weight of the truss is not taken into consideration [1,2].

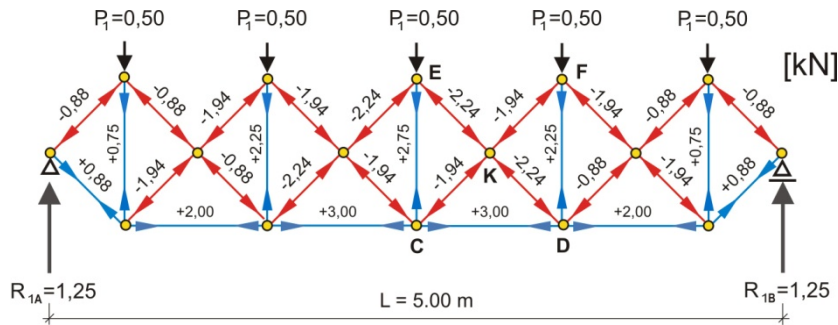


Fig. 3. Force values calculated in the first stage

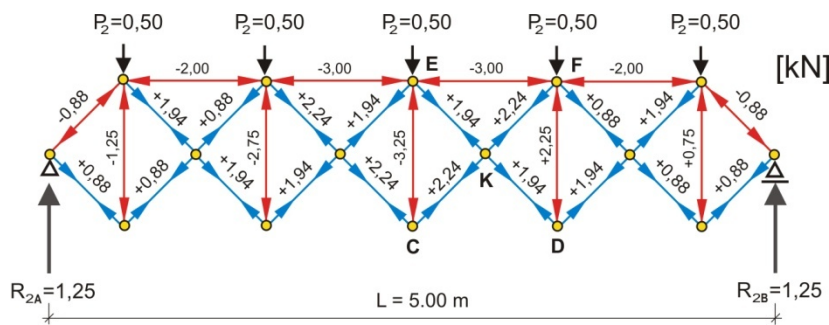


Fig. 4. Force values calculated in the second stage

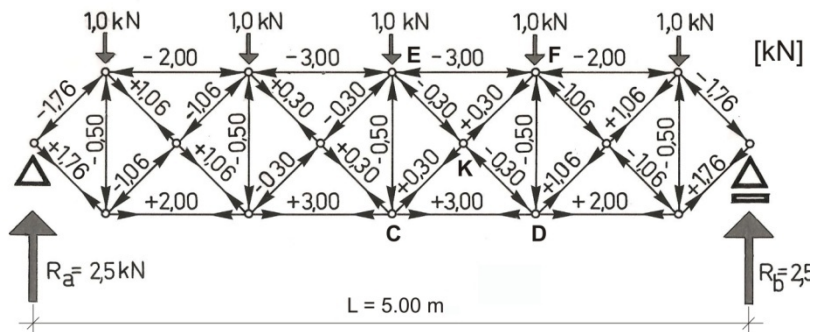


Fig. 5. Force values calculated in the basic truss as a result of appropriate application of principle of superposition in the proposed two-stage method of calculation of statically indeterminate trusses

One should be aware that the two-stage method gives in result the approximate values of forces because each simple method applied in the both independent stages, like e.g. the Cremona's method, does not take into consideration differences between stiffness of members joined to the same nodes [3,4]. Simplicity of the two-stage method is caused by suitable application of rule of superposition. The feature can be considered as the most important value of this method that is why it can be especially useful for the preliminary and even for the very advanced calculations of the statically indeterminate trusses based on numerical models of such structures defined in appropriate programming language.

## 2. Structural concept of system of combined foundation

Principle of superposition has been also applied during invention of structural system of combined foundation [5]. Very large surface of the foundation can guarantee that the stress value in subsoil will not exceed permissible value of the subsoil load ability. Increasing the foundation surface has to be made with respecting the basic rules of theory of structures and engineering design of foundation systems for e.g. tall buildings [6]. The point of this structural system is to transmit the outside big force  $V$ , see Fig. 1c-d, by means of suitable nodes of an intermediate structure, to the matter of basic components shaped in form of e.g. beams (1) located on a common horizontal base. The load force  $V$  is applied to the upper node A of a short strut AB, which is inserted tightly inside appropriate guides and which has only one degree of freedom along the vertical direction. To the lower node B of this short vertical strut is connected the intermediate structural system composed of two independent parts.

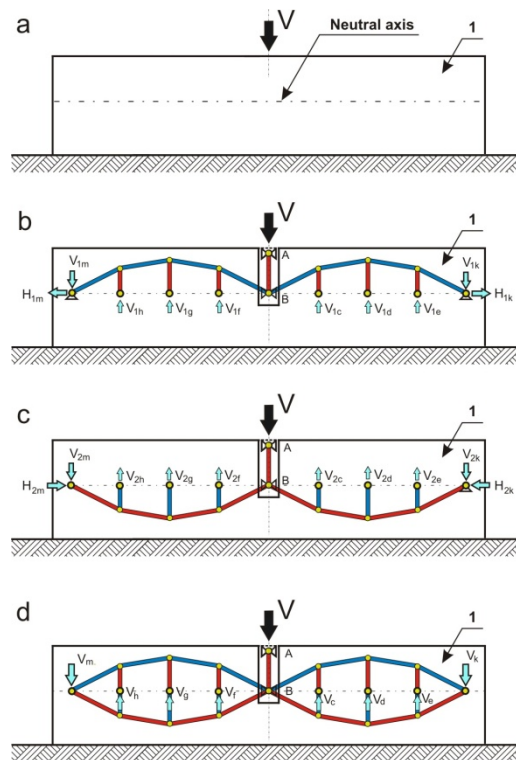


Fig. 6. Analytical schemes of structural concept of system of combined foundation

Scheme of its upper part is shown in Fig. 6b while scheme of its lower part is presented in Fig. 6c. Patterns of both parts are symmetrical towards the horizontal central axis of a beam, where moreover are placed nodes connecting these parts to the main bodies of the beams. After combining both parts, see Fig. 6d, the horizontal components of reactions are wiped out because shapes of both parts of the intermediate system are symmetrical towards the horizontal central axis. Final lenticular shape of the intermediate system can be put in the narrow space between two parallel beams, or it can be suitably arranged around the body of a single beam.

Lenticular segment, consisting of all appropriate component parts, can be considered as the structural unit of foundation system shaped in this way. If suitable bracing system, having form of a kind of lenticular girder, is suitably arranged in structure of the aboveground stories and multi-story building is supported on the above described foundation, see Fig. 7a, then the whole structure is called the combined system of tall building [7]. Number of replication of lenticular units along horizontal direction is optional, see Fig. 7b, what implies that horizontal surface of that foundation system can be very large, see Fig. 7c, and theoretically it can be unlimited. The proposed system of foundation makes possible the safe location of even very heavily loaded building on subsoil of very small load carrying capacity.

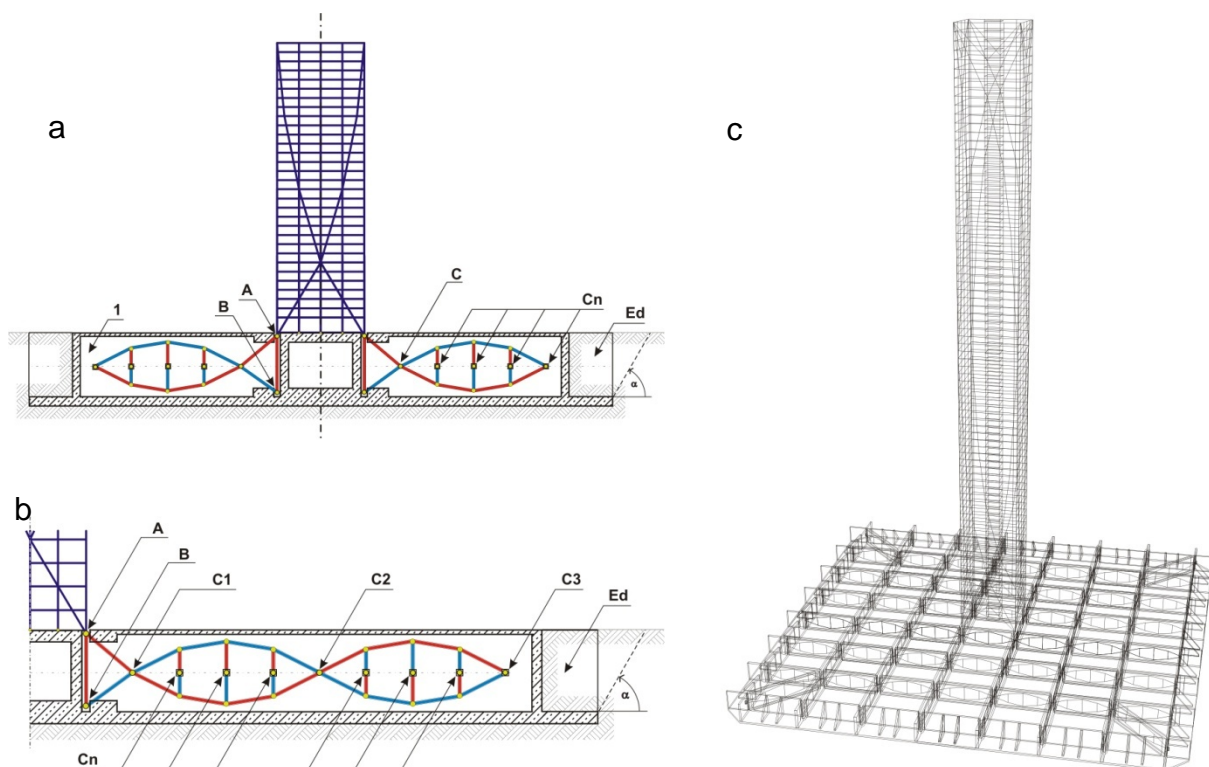


Fig. 7. Example of forms and applications of system of combined foundation

Design process of such complex structural system is also complicated, difficult and it takes a long time. Efficiency of the design process is considerably enhanced by application of numerical models defined e.g. in the programming language Formian [8]. It makes possible to use parameters, which applications make the process very flexible. Numerical models, defined by suitable sets parameters, enable the fast and easy modifications of shape of the designed building, which often has to be adjusted to the current requirements of the investment process.

Below there is presented text of Program F, prepared in programming language Formian, which defines numerical model of structure presented in Fig. 7.

#### Program F

```

01. use &,vt(1),c(1,1),vm(2),vh(0,-40,26,0,0,0,0,1);
02. Belp1=[{0,0,0;40,0,0],[40,0,0;40,0,10],[40,0,10;0,0,10]};(*)Foundation beam(*)
03. Plyta=[{0,0,0;40,0,0],[40,0,0;40,0,-1.5],[40,0,-1.5;0,0,-1.5]};(*)Base slab(*)
04. ScianOp=[{34,0,0;34,0,10],[33,0,0;33,0,10]};(*)Retaining wall(*)
05. LiniaS=[0,0,5;40,0,5];(*)Central axis of foundation(*)

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- 06. Os1=[6,0,1;6,0,22];(\*)Axis of outer column(\*)
- 07. Budynek={ [6,0,13;0,0,13], [6,0,10;0,0,22], [0,0,16;6,0,16], [0,0,19;6,0,19],  
[0,0,22;6,0,22], [0,0,10;0,0,22], [3,0,10;3,0,22]};
- 08. TrojkGl={ [6,0,1;11,0,5], [11,0,5;6,0,10]};
- 09. Soczewka=lam(3,5)|lam(1,21)|{ [11,0,5;16,0,8], [16,0,8;21,0,9],  
[21,0,9;21,0,5], [16,0,8;16,0,5]};
- 10. Tuleja={ [6.35,0,0;6.35,0,10], [5.65,0,0;5.65,0,10], [0,0,1.5;5.65,0,1.5], [0,0,9;5.65,0,9]};
- 11. PlytaGor={ [0,0,10.5;6,0,10.5], [6,0,10.5;34,0,10.5], [34,0,10.5;34,0,10],  
[34,0,10.5;40,0,10.5], [40,0,10.5;40,0,10]};
- 12. Bloki={ [6.35,0,9;9.25,0,9], [9.25,0,9;9.25,0,10], [6.35,0,1.5;9.35,0,1.5],  
[9.35,0,1.5;9.35,0,0]};
- 13. PolowaF=Belp1#Plyta#ScianOp#LiniaS#Os1#Budynek#TrojkGl#Soczewka#Tuleja#PlytaGor#Bloki;(\*)Half of the foundation(\*)
- 14. CalyFund=pex|lam(1,0)|PolowaF;
- 15. StropyG1=pex|lam(1,0)|rin(3,12,3)|[0,0,25;6,0,25];
- 16. StropyG2=pex|lam(1,0)|rin(3,15,3)|[0,0,14.5;6,0,14.5];
- 17. Parabol1=pex|lam(1,0)|{ [0,0,22;3,0,34], [3,0,34;4.5,0,46], [4.5,0,46;6,0,58]};
- 18. SlupyB=pex|lam(1,0)|rin(3,3,12)|rin(1,3,3)|[0,0,12;0,0,34];
- 19. TallBudA=StropyG1#CalyFund#Parabol1#StropyG2#SlupyB;
- 20. draw TallBudA;(\*) See Fig. 7(\*)

## 2. Example of possible application of system of combined foundation

When particular components parts will be constructed as the waterproof boxes then the system of combined foundation will have a huge hydrostatic lift. That is why it may be considered as a kind of an artificial floating island and it could be the base for a building located there, see Fig. 8 and Fig. 9. The Ocean Agave is planned as a relatively small and independent settling unit that is self-sufficient in terms of energy and food supply. The conceptual design was prepared for needs of international architectonic competition eVolo2015 (author: Janusz Rębielak; technical cooperation: Wojciech Kocki and Maciej Rębielak). It is designed as an artificial island free-floating in subtropical ocean areas, in the far distance from land, and able to house a minimum of 120-150 persons.

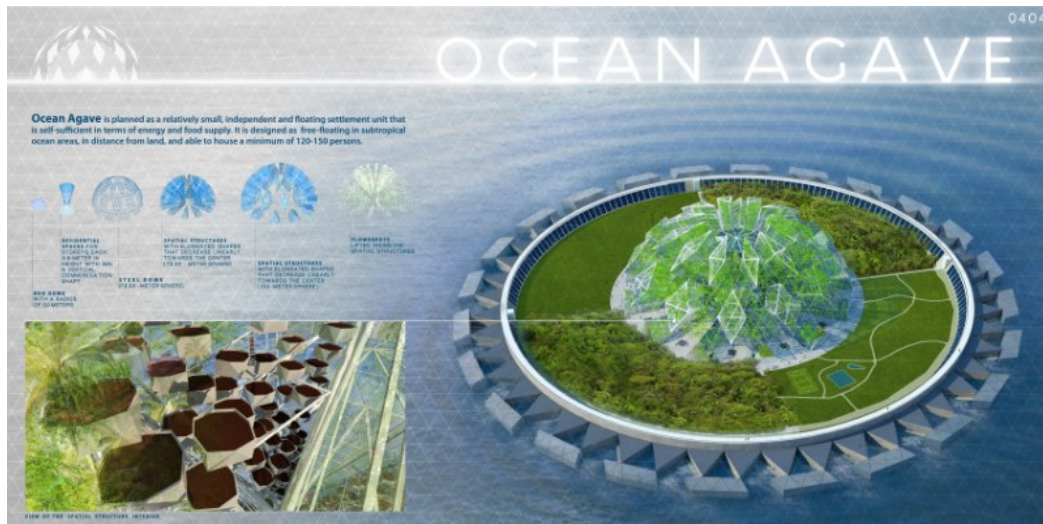


Fig. 8. Bird view of the floating platform called Ocean Agave

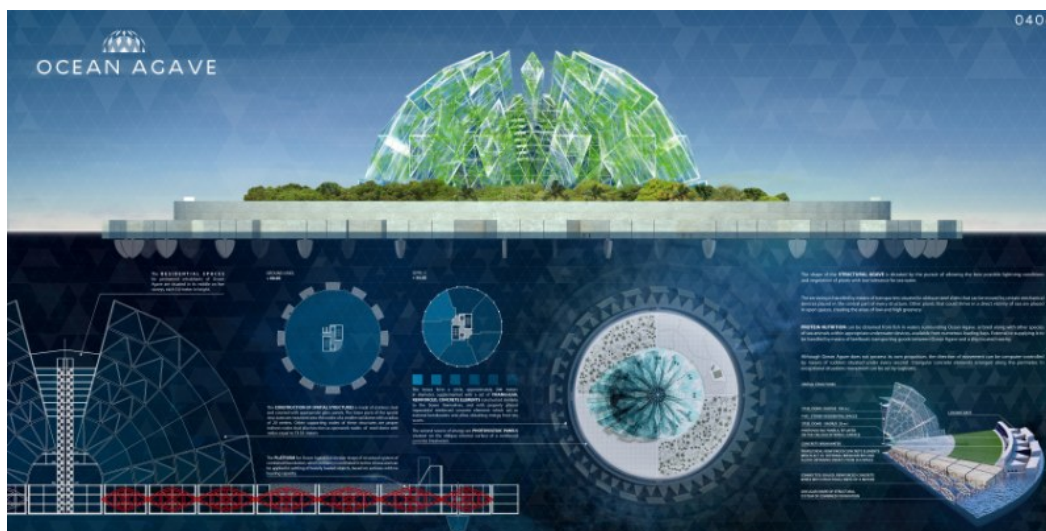


Fig. 9. Analytical schemes of the Ocean Agave

The platform for Ocean Agave is a circular shape of the proposed system of combined foundation, which in theory is unrestricted in terms of area and can be applied in settling of heavily loaded objects, based on surfaces with no bearing capacity. It is created using properly connected, sealed, reinforced concrete crates with structural depth of 15 meters, and with substantial uplift pressure. The draft of the designed structure is estimated between 2.50 and 3 meters. The crates form a circle, approximately 400 meters in diameter, supplemented with a set of triangular, reinforced concrete elements, constructed similarly to the crates themselves, and with properly placed trapezoidal reinforced concrete elements which act as breakwaters and allow obtaining energy from sea waves. The center of the base houses is a complex structure called Agave, divided into several dozen spatial structures with elongated shapes that decrease linearly towards the center, and whose axes converge in the central point of the whole setting. Their cross-sections are mostly rhombus-based, while only these directly connected to the upper platform's surface have their cross-sections shaped into triangles. A structure built this way possesses a form that closely relates to a sphere of 100-meter radius. The construction of spatial structures is made of stainless steel and covered with appropriate glass panels. The lower parts of the spatial structures are mounted onto the nodes of a smaller rod dome with a radius of



20 meters. Other supporting nodes of these structures are proper indirect nodes that also function as openwork nodes of a steel dome with a radius of 73.33. meters. The main residential structure is as tall as the before-mentioned radius, and is situated vertically over the central point of the structure.

### Closing remarks

Appropriate application of principle of superposition makes possible to find new and very effective methods of calculations as well as the types of structural systems. Numerical models of architectonic buildings and of their structural systems, generated by application of selected sets of parameters and by means of suitable programming languages, make the design process fast and very efficient.

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