RANDOMNESS, DIS(ORDER), AND GENERATIVITY

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Abstract

One of the most used, but maybe most controversial, terms, when one refers to generative art, is randomness.
In "Chance and Generativity" (GA 2008), chance (randomness) was examined relatively to generative processes, and mainly considered along the huge possibilities of combinatorics and the necessity of arbitrarily (and then randomly) choosing among results, which are all, though not strictly *identical*, definitively *similar*.
Here we explore randomness with its links to order and disorder, and its capacity to generate forms or structures.
First, one needs to objectively define those terms (*randomness*, *disorder*), along with others like *entropy*, *complexity*, and confront their reality, which can be measured as probabilities, to our perception of them. For instance, *interacting* randomness may lead to something which seems more ordered, though actually it is not (*cf* Jennifer Galanis and Martin Ehler, "Disorder disguised as Order", GA 2011). This topics can be illustrated in many ways.
Those illustrations of randomness comprise constrained packings (of rods, circles, …, which must not overlap) and the more or less ordered structures they generate, according to the rules one gives oneself in the packing.
Some rules may be interpreted as behaviours. Then we can see how more or less *random* behaviours (erratic walk especially) may paradoxically generate forms that are ordered at some level.
All those experiments illustrate phenomena like emergence, spontaneous patterns, crystallisation, and so on, and let us think about the links between nature, science, and art.

Introduction

According to wikipedia, “randomness means lack of pattern or predictability in events. Randomness suggests a non-order or non-coherence in a sequence of symbols or steps, such that there is no intelligible pattern or combination.” [1]

Randomness may apply to very different situations, and we will not discuss it abstractly, but in a defined context: some configuration of some shapes, in some 2D space (possibly extensible to a 3D space).

We first need to define that space, and the principal distinction is between a *discrete* and a *continuous* space. If one wants to distribute *random* points in a square, for instance, there is a difference between choosing random pixels in a bitmap (discrete space) and choosing random points in a square that is a subspace of R^2.

In a discrete space, it is a question of combinatorics: there may be a huge number of different configurations, but their number is finite. The probability of some configuration is the proportion of this configuration among all possible configurations, i. e. 1 divided by the number of different configurations, which is a finite number. For instance, the number of 10x10 bitmaps, where pixels may be white or black, is 2^{100}. This is a very large number ($\approx 10^{30}$), but not infinite. Any configuration of pixels in such a bitmap has a probability of $1/2^{100}$.

Defining the probability of an event as the proportion of some event(s) among all events (which you can count) could infer that any probability is a rational number, but it is not so. In a continuous space there is an infinite number of configurations, and you cannot calculate a fraction with ∞ as denominator, but you can still calculate probabilities. For instance, drawing a disk of diameter d inside a square of side d, you can calculate the probability for a point randomly chosen in the square to belong to the disk by dividing their respective areas: $\pi d^2 / 4d^2 = \pi/4$, which is not a rational number. Neglecting his dear naturalism for a while, but in the interest of games of chance, popular at his time, Buffon proved that if you throw a needle of length 2a on a surface where parallel lines are drawn with an interval 2b, the probability that this needle intersects one of the line equals $2a/\pi b$.[2] [3].

A random distribution of points in a square of side d, which is a subspace of R^2, is one where you choose their coordinates anywhere in [0,d]. There is no way to determine the probability of any configuration, but if you divide the square into a grid, the probability of each point to be in any box of the grid is the same. That is, if you have a great number of points, there will be approximately the same number of points in each box. And it must be true for any size of the grid.

Beyond randomness, what we are interested in is to try to define *order* versus *disorder*, and what our perception detects as a *pattern*, something which we find “interesting”, versus configurations which are indifferent. It is also dependant of the context, and we will try to objectively define what an interesting pattern is in each context. It is generally something between total order and total disorder, which are both without interest. It is opposed to randomness which supposes a lack of pattern, but randomness may be an initial condition in a process that leads towards some pattern. Those patterns, which are neither totally ordered nor disordered, but contain an underlying order more or less broken by a certain amount of disorder, are what we qualify as "(dis)order".

We support our discussion with two models, one that works in a discrete space, and one that works in a continuous space.

I. Randomness and (dis)order in a discrete space: around the Ising cellular atomaton

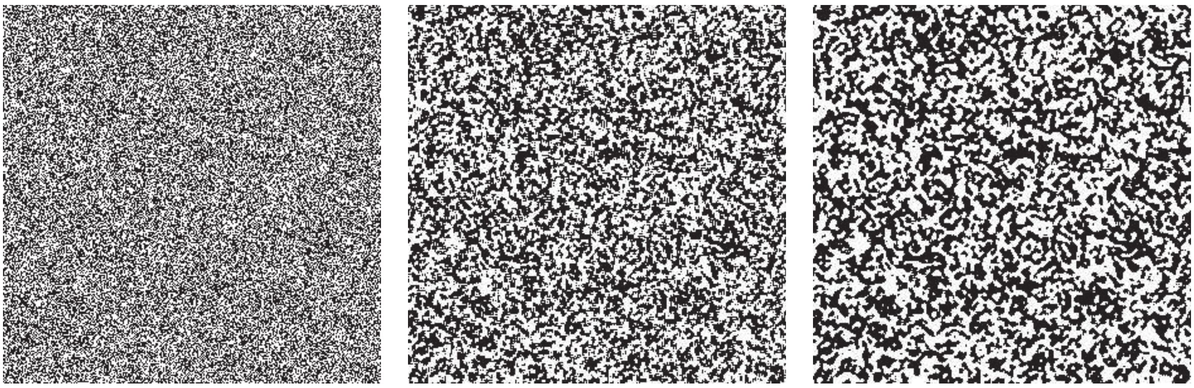
I.1. The Ising cellular automaton

Cellular automata work in a discrete space, generally an orthogonal grid, which we can easily assimilate to a bitmap of pixels. All cellular automata evolve according to the neighbourhood of each cell, either the four neighbours at the edges, or the eight neighbours obtained with adding the ones at the corners. Cells may have any number of states (visualised as colours), but there are already a great lot of results to consider with two-states CA (0 and 1). Metaphorically, one can say that cells are dead or alive. The state of each cell evolves according to the states of its neighbours, but then again, there are a lot of different results when considering only the sum of those states, what one calls "totalistic" CA. The famous "game of life" is one of those totalistic two-states CA.

A CA "works" well when it evolves and does not disappear too fast, i. e. when all cells do not "die" immediatly. This may depend upon the initial conditions. In the game of life many configurations have been studied, some die rapidly, some others after some generations, others seem to evolve indefinitely.

There is a totallistic two-states CA that simulates an actual physical behaviour: the Ising model [4]. The two states do not refer here to life and death, but to the magnetic moments of atomic spins that can be in one of two states (-1 or +1). In this model, each cell adopts the state of the majority of its neighbours (the "majority vote").

Notwithstanding the scientific side of this model, it interests us here for its involving randomness, order and disorder, in ways we want to discuss. A typical evolution of such a CA is shown in fig. 1. In all these experiments, we consider a "torical topology" (periodic boundary conditions).



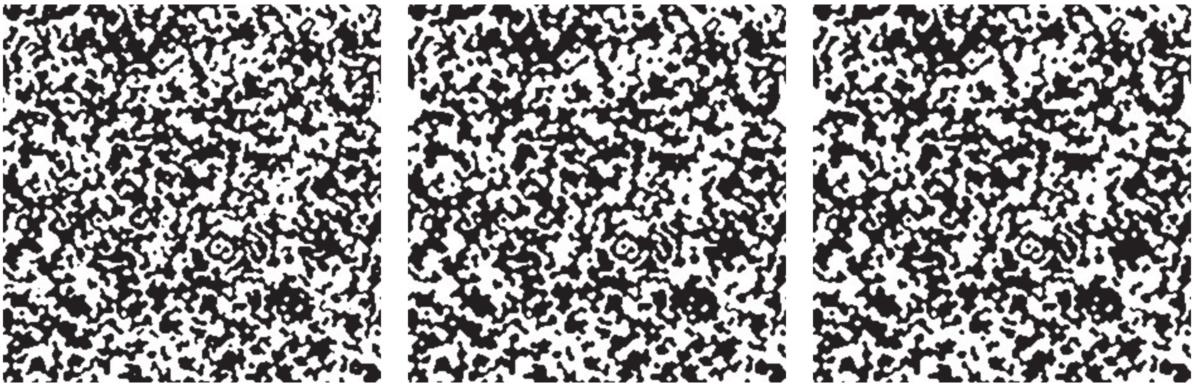


Fig. 1: typical evolution of Ising CA (300x300)

This CA may use different “depths” of neighbourhood, by not only considering the 8 cells lying directly around each cell, but also the 16 ones around them (depth 2), and even the 24 ones beyond (depth 3), and so on. It works as a sort of zoom on the process, and the patterns are similar, but larger.

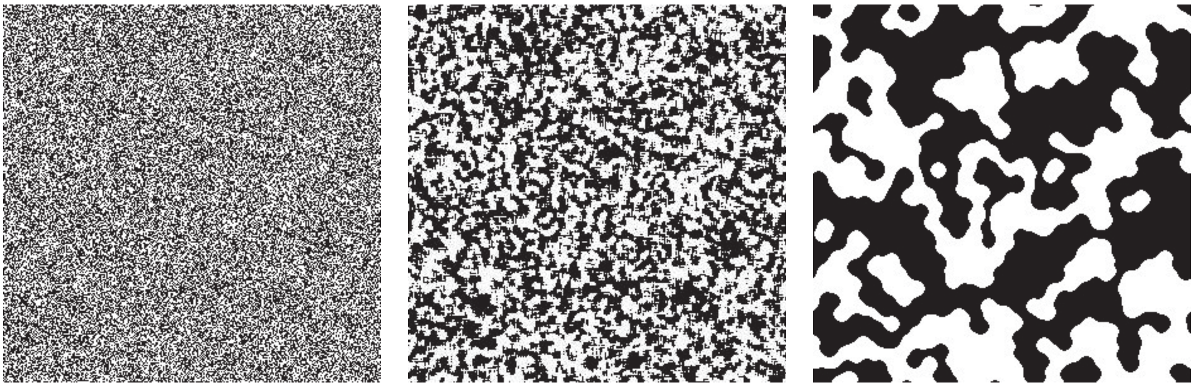


Fig. 2: Ising CA (300x300) with depth 2

What makes us recognise the result as a “pattern”, as “(dis)order”, is, in a first approach, that it seems not totally predetermined (each pattern we obtain is different) but not absolutely nondescript either (like the random initial distribution). We will try to measure what has changed during the process later, but here we can say that those patterns show smooth interlocking of black and white areas, with boundaries that curve themselves in a “natural” way. Without being figurative at all, it can remind us of natural patterns. And, as it were, some animals, horses and cows (especially one cow breed native to our region, the “Bretonne pie-noir”), and other ones, called “piebald” or “pied”, display such patterns...

I.2. Randomness and balance

This CA is supposed to start with a “random” and balanced distribution. The condition of being balanced seems straightforward: if one state predominates at the start, with the majority vote, it is indeed likely that this state will spread and invade all the automaton. But we will see later that it is not such a strong condition.

Let us discuss the “random” condition of the starting distribution. In order to obtain such a random balanced initial state of the automaton, one considers each cell, and “flips a coin”, i. e. chooses randomly between -1 and 1 (or 0 and 1, or black and white). One typical configuration is such as fig. 3.

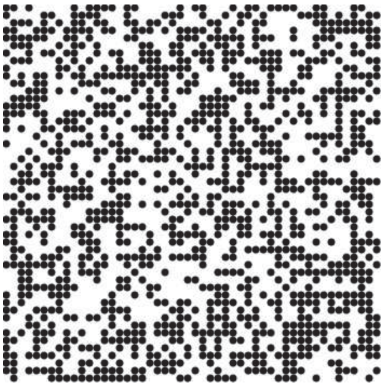


Fig. 3: typical initial state of Ising CA (50x50)

The CA evolves rapidly (in 20 steps) towards a fixed state (Fig. 4).

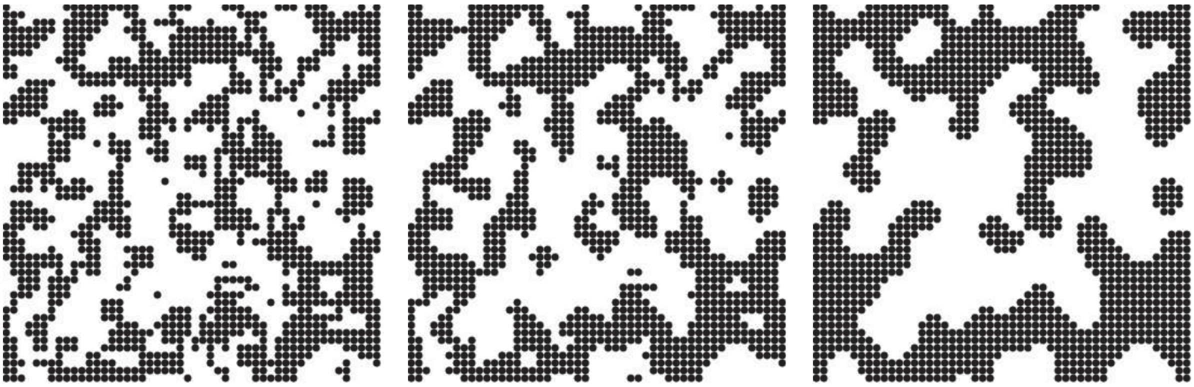


Fig. 4: steps 1, 2 and 20 of Ising CA (50x50)

The automaton has got 50x50=2500 cells. 2500/2=1250, so there should be around 1250 black cells at the start. Actually in this particular example, there are 1242 black cells. It does not perturb the evolution, though. Through the steps, the count of black cells evolves around 1250, and stabilises itself at 1264:

1242 1227 1236 1248 1249 1252 1254 1251 1248 1247 1248 1250 1251 1252 1256 1258 1261 1262 1263 1264 1264 ...

A more subtle way of obtaining a balanced random distribution is to consider the statistical density of black cells. A balanced distribution corresponds to a density of 0.5. We can then try the plasticity of the starting conditions by choosing densities different from 0.5.

With a density of 0.75 and more, the automaton becomes all black. But with a density of 0.7, though a lot of evolutions have the same fate (all black), some keep one, or even two small white area(s) (fig. 5):

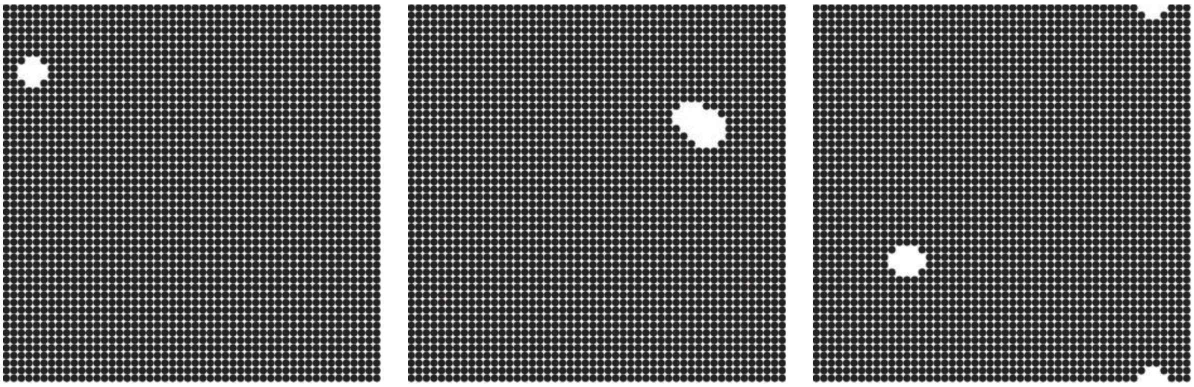


Fig. 5: some final results of Ising CA (50x50) with density 0.7

Below a density of 0.7, white areas remain allways. Typical results are shown fig. 6:

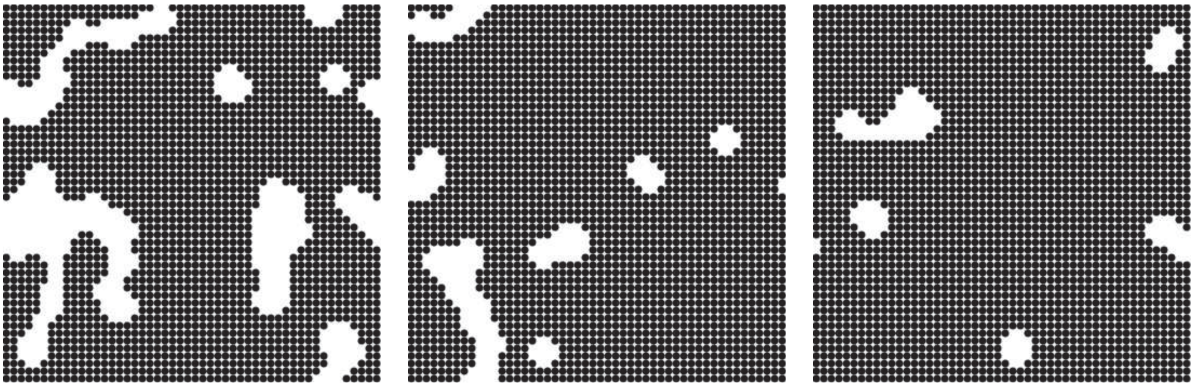


Fig. 6: typical results of Ising CA for densities 0.55, 0.6, 0.65

This CA and its starting conditions allow us to better define randomness and disorder. If we start with ordered though balanced starting conditions, the CA does not work. A strict equality between black and white cells happens in ordered configurations such as striated and checked patterns. A striated pattern evolves as a “blinker”: the black and white stripes exchange themselves at each step. A checked pattern does not evolve, it remains unchanged.

Randomness is then crucial to this CA, more than balance of states of cells.

I.3. Randomness, (dis)order and neighbourhood

In order to better define this randomness, and the disorder it contains, we can highlight a property of cells, directly involved in the evolution of the CA: the number of neighbours. Fig. 7 shows the “black” cells coloured according to their number of “black” neighbours, from 0 to 8 (from red to purple), and the number of cells that have those numbers of neighbours:

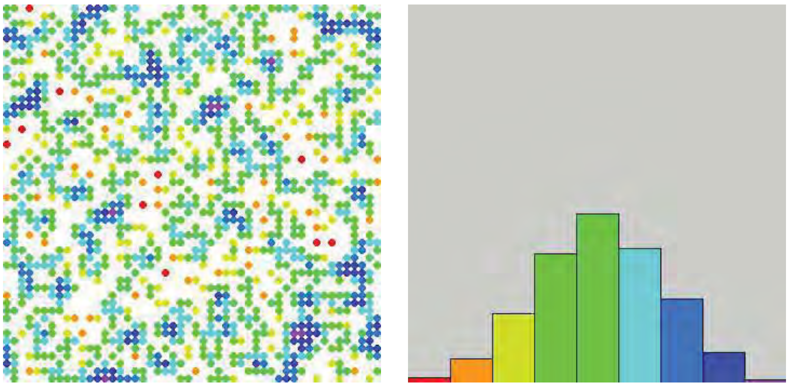


Fig. 7: "black" cells coloured according to their number of "black" neighbours, and amounts of "black" cells by numbers of neighbours

One can see that the repartition of the numbers of neighbours shows a nice so-called "normal" distribution, or gaussian curve. It is a good characterization of a balanced "random" configuration in this context. It is important to remark that we could do the same visualisation for the white cells, which would show the same repartition.

The process of the CA consists in destroying this normal distribution (fig. 8):

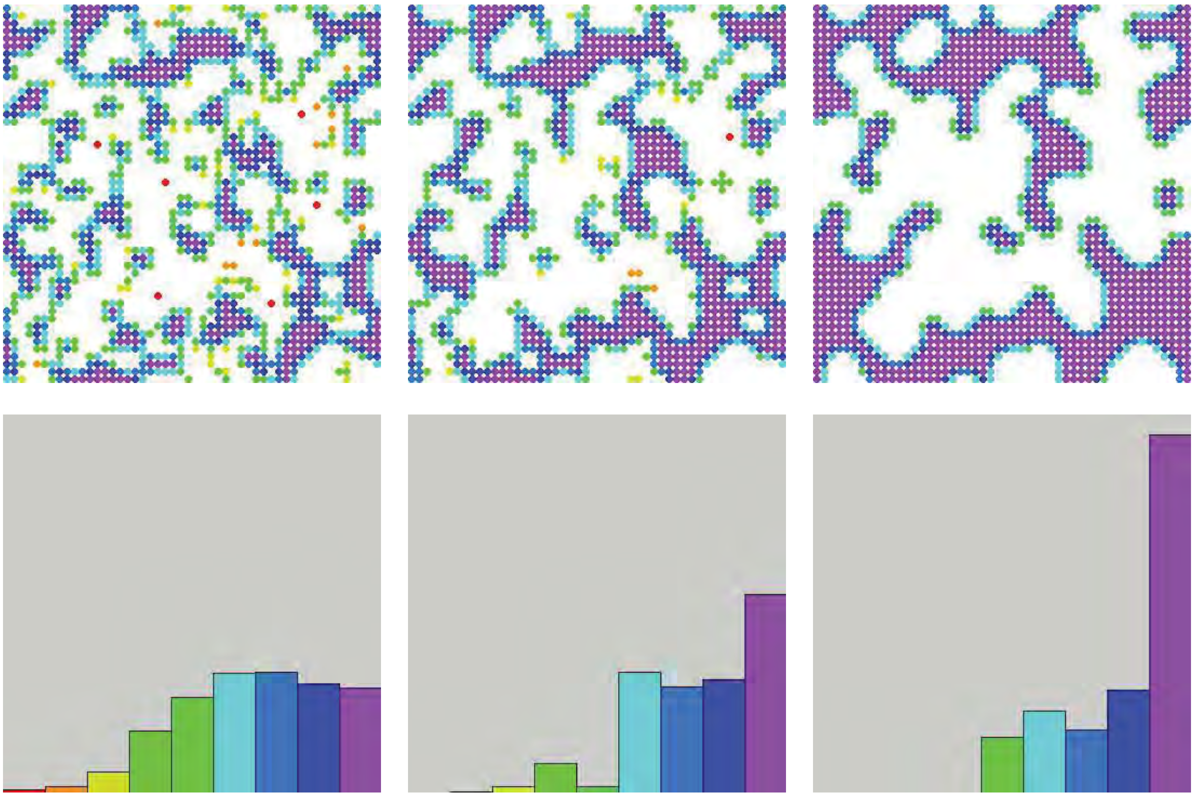


Fig. 8: numbers of neighbours (0 to 8) for steps 1, 2 and 20

By contrast, we can analyse the ordered configurations with the same criteria. The stripes contain black cells that have all only 2 black neighbours; the process of the CA turns those cells white, but the white ones are in the symmetric situation, so they turn black. Hence the “blinker”. For the checked configuration, any cell, white or black, has 4 neighbours in the same state, so the situation is blocked in the nest.

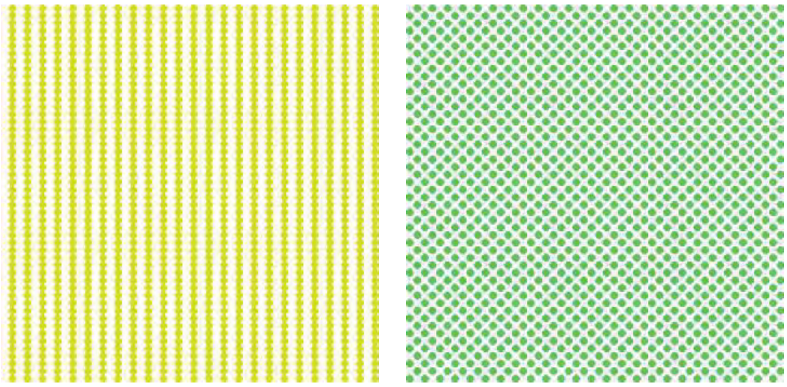


Fig. 9: numbers of neighbours for two ordered initial stages of Ising CA

One other consideration about this model is that we have seen that the balanced condition was not a very strict one when cells were distributed randomly. Now, if we introduce only one perturbation in the checked (and stable) initial stage, the “error” spreads, and invades all the space.

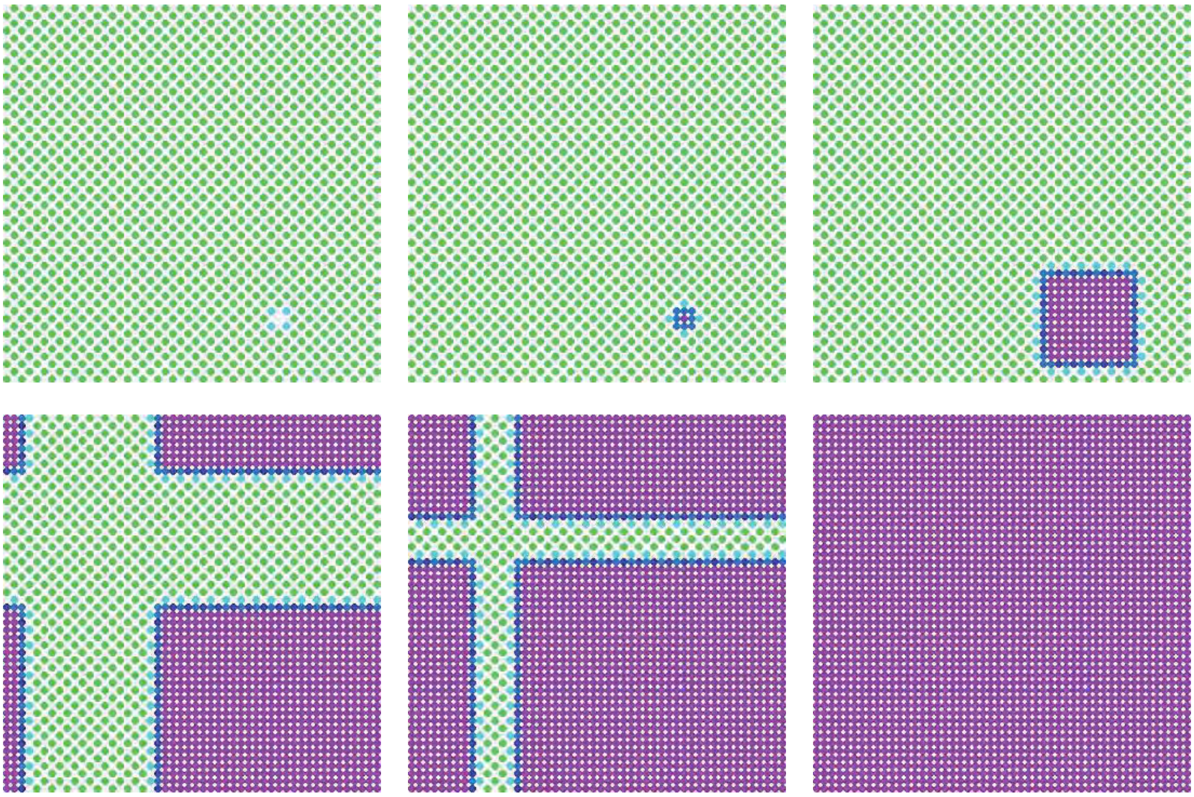


Fig. 10: evolution of Ising CA from initial checked configuration with one error

Leaving alone the Ising CA now, let us examine random densities according to numbers of neighbours.

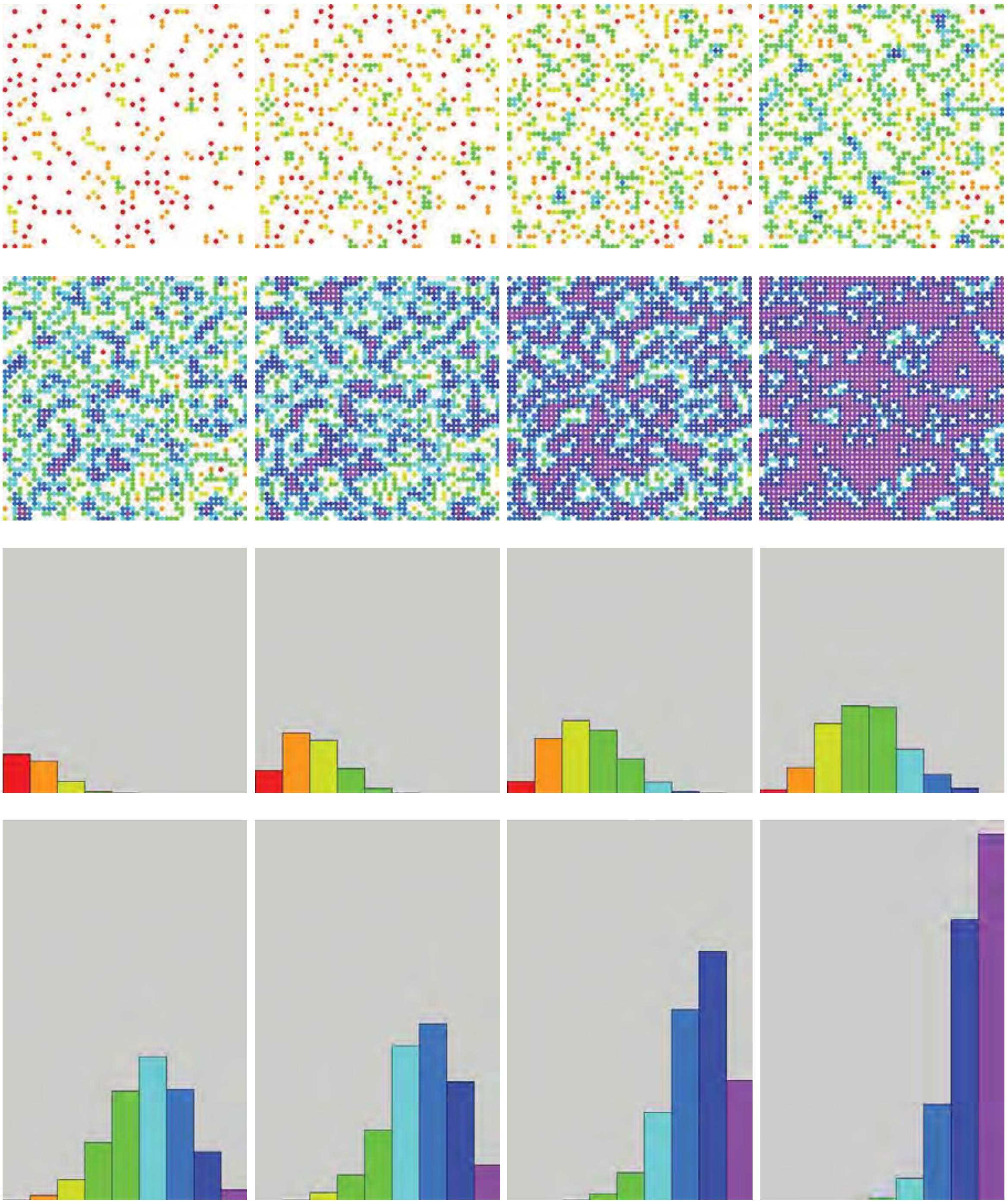


Fig. 11: numbers of neighbours for random densities 0.1, 0.2, 0.3, 0.4

It seems that with a high density (0.8 and 0.9) we have a distributed neighbourhood that resembles that of the results of the Ising CA. Examining the amounts more precisely, there is some difference though. In Ising results we had a predominance of 8-neighbours cells, but 4-, 5-, 6-, and 7-neighbours cells remained. In random density 0.9, 8-neighbours cells predominate, there are no more 4-neighbours ones, and 5-, 6-, and 7-neighbours ones are not of the same amount. We see that it tends progressively to the 1.0 density where all cells have 8 neighbours.

The other important difference is that in the Ising CA results the repartition of white and black cells (and their neighbourhoods) remains balanced. It is this balanced distribution of black and white cells, but also of their neighbourhoods, that characterises Ising patterns.

The discussion could have been continued about cellular automata in general but we will now address another model, which works in a continuous space, though we will see that this question of neighbours will also play a key role.

II. Randomness and (dis)order in a continuous space: disk packing

II.1. First approach

We became aware of this topics through the article "Spontaneous Patterns in Disk Packings" [5] to which we owe a lot in this part. Problems of packing, in general, consist in looking for some configuration of shapes in some space that maximises the density of those shapes, i. e. the proportion of space occupied by those shapes.

Packing disks in a certain area of the plane has been much studied. One must first notice that we can easily define an ideal optimised packing, theorically unlimited (without any boundary), and that it consists in a triangular grid of disks, each one occupying a hexagonal subspace. So the ideal density is easy to calculate: the area of a disk of radius 1 equals π, and the area of the hexagon in which the circle is inscribed is 6 times that of the equilateral triangle of height 1, i. e. 6 times $1/\sqrt{3}$. So the ideal density equals: $\pi\sqrt{3}/6=0.90689968...$

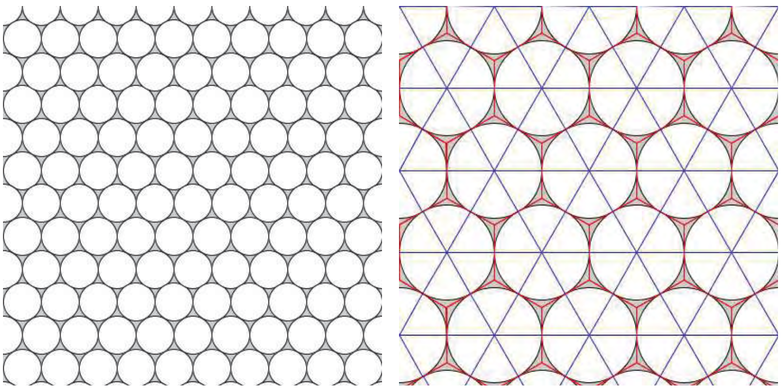


Fig. 12: ideal packing of disks

When boundaries come into play, it is not so simple... If the ideal packing had been orthogonal, there would not have been any problem because its dual is also othogonal, and the square is self-similar, i. e. you can pack squares inside a square. Even if the ideal grid had been hexagonal (each subspace being a triangle), there would have been no problem either, because the equilateral triangle is self-similar as well, and one could have packed "disks" with a maximal density in a triangular area.

But, as it were, the hexagon is not self-similar... So, even with an hexagonal area, never will we be able to pack disks with the maximal density we defined previously.

The hexagon cannot tesselate any convex polygon, though it tesselates the illimited plane; that is a weird but insurmontable characteristics of 2D space...

II.2. Physical model

Along with digital simulations, we wanted to examine the physical behaviour of disks, more exactly balls, or marbles, on a plane. We chose small balls used for toy pistols, for their claimed regularity of diameter, and smoothness of surface.

A first observation is that if you pour such balls into any plane recipient, as soon as there are enough of them, they tend to form ordered areas, separated by more disordered boundaries. These first experiments were not very rigorous, so we built a machine designed to pack balls.

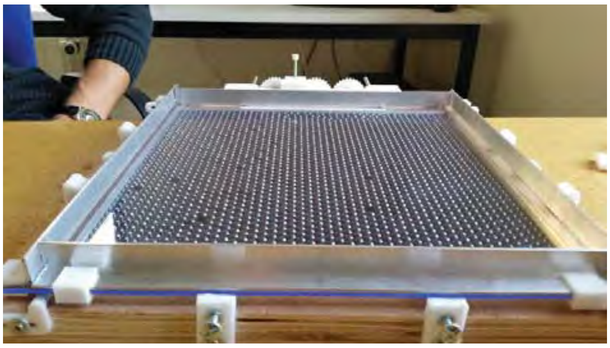


Fig. 13: packing-ball machine

This machine consists in a frame, one side of which is pushed by a system of cogwheels and screws. This moving side packs the balls progressively. We put a plexiglass sheet over the balls in order to help them from moving up. As it were we observed that forces are so strong that, in spite of this precaution, the balls had the tendency to push up on the plexiglass... The desire for an extra dimension of space is very pregnant...

Anyway, we had some results with this device as such. We saw ordered areas forming, and holes (one ball missing), and boundaries that went from curved to straight ones, often perpendicular to the applied force, and phenomena that will appear also in the digital model, such as triangles, and so on.

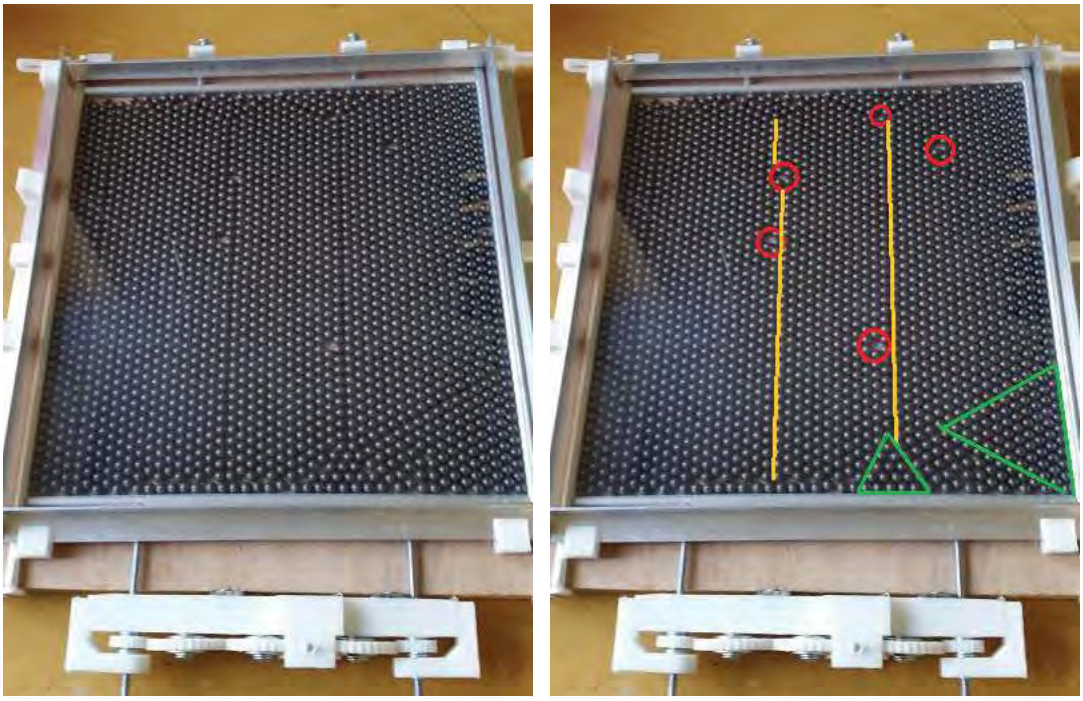


Fig. 14: holes, straight lines and triangles appearing when packing balls

II.3. Digital simulation with mechanical laws

A preliminary remark about a digital simulation, is that we cannot pretend that it works in a *continuous* space. Obviously, no digital model can achieve such an ambition. Real numbers are only approximated by computers, and any digital space is discrete. Hopefully the high resolution of that space manages to simulate a continuous space.

Inspired by [5], the model consists in starting with a random distribution of points, considered as disks with diameter 0. The disks are growing, and as soon as two disks happen to overlap, a force is applied to their centers in order to part them. This system simulates a shrinking of space where a certain number of disks (with a fixed diameter) are first randomly distributed. Contrarily to our physical model, there is no predominant direction of forces.

The growth of the disks happens by discrete steps (0.01 at each step for instance). A crucial element of the process is the rate of this growth. The slower the disks grow, the more compact the final configuration will be.

We considered a square space with borders, or with periodic boundaries. We did some experiments with borders first, and obtained results showing areas seemingly ordered (compact and on a triangular grid), separated by chords issued from the corners (fig. 15):

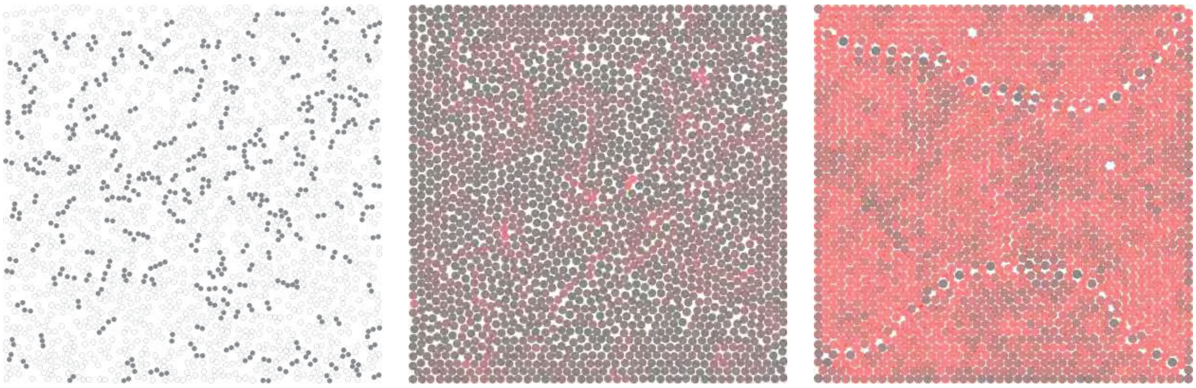


Fig. 15: packing with borders, 2000 disks, steps 220, 400, 1040

We colour the disks according to the forces applied to them (white (no contact at all), and from grey to red). This allows us to see the blocked disks (in red), and those that are more relaxed. The background is also white: the three white “stars” in the final picture of fig. 15 are holes, where one could place a disk. We see that in that final stage, there are three roughly compact areas separated by boundaries which we remark because we see the white background. In these boundaries, disks are mostly subject to smaller forces. But in the compact areas some are not totally blocked either.

We mostly started from random distributions, but we also wanted to see what happened with regular initial distributions. We put the initial 0-diameter disks on a triangular, orthogonal, and hexagonal, grid. A issue is the shape of the frame, either a square or not. We can keep the square with the orthogonal grid, but with the triangular and orthogonal grids, we cannot, since their horizontal and vertical spacings are not the same: they are in the proportion $v/h=\sqrt{3}/2$. With the condition of the periodic boundaries, the initial configuration must repeat itself horizontally and vertically exactly. So the initial frame is a rectangle, not a square.

Not unexpectedly, the triangular and orthogonal grids are stable, since all disks are blocked by their neighbours, but the hexagonal grid collapses, and has the same ulterior fate as a random distribution (fig. 16):

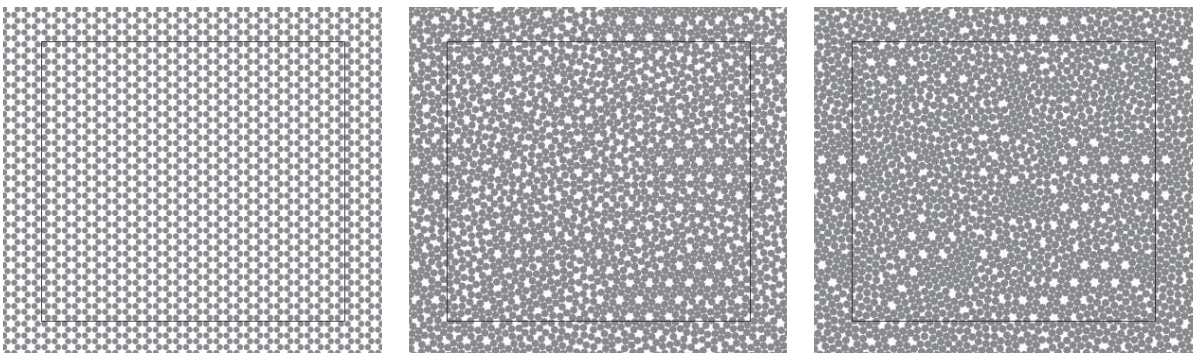


Fig. 16: collapse of hexagonal distribution of disks

What is interesting is what happens with a slightly perturbed triangular or orthogonal grid. One added random disk is enough to perturb the stability of the system.

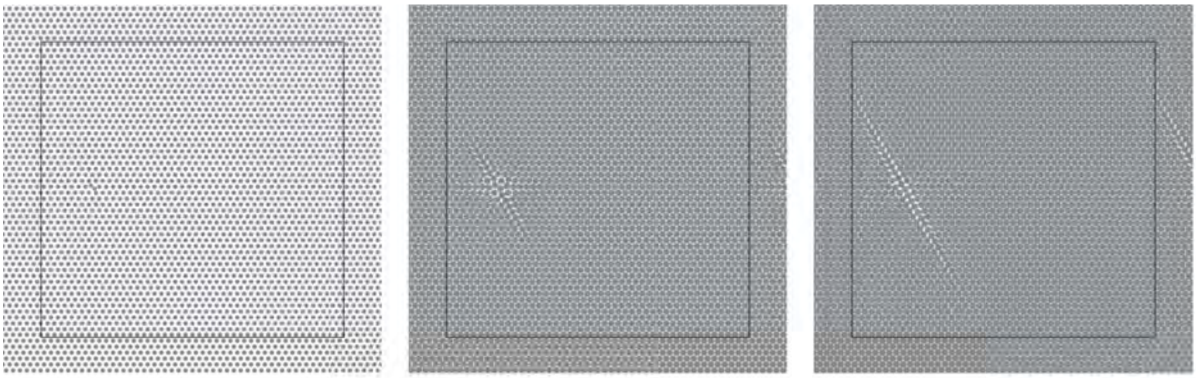


Fig. 17: triangular grid with one added random disk

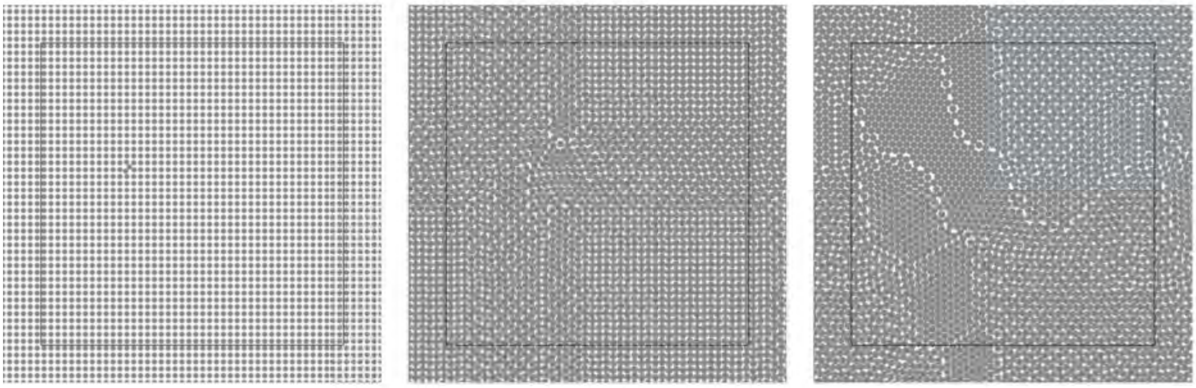


Fig. 18: orthogonal grid with one added random disk

In this last case, the configuration becomes similar to those we get from initial random distributions. We remark that the most stable configuration (triangular grid) is less perturbed, but is perturbed anyway, which is an indication of the difficulty of getting a regular configuration starting from a completly random initial distribution.

Going through all the results we obtained, we retained two categories: “crystals”, the most compact ones, very strained though not completely ordered, and “wallpapers”, patterns which the periodic display and the colours chosen in order to show different properties of the disks, induce inevitably such an image... In this last category, we got sometimes rough stripes, horizontal, vertical or oriented at 45°.

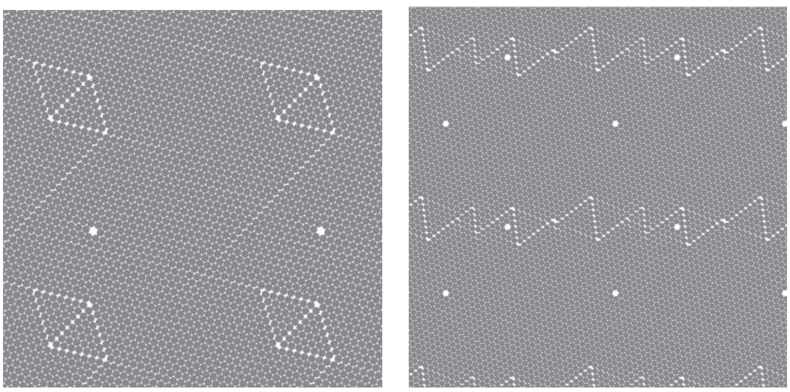


Fig. 19: “crystals” (2x2 display)

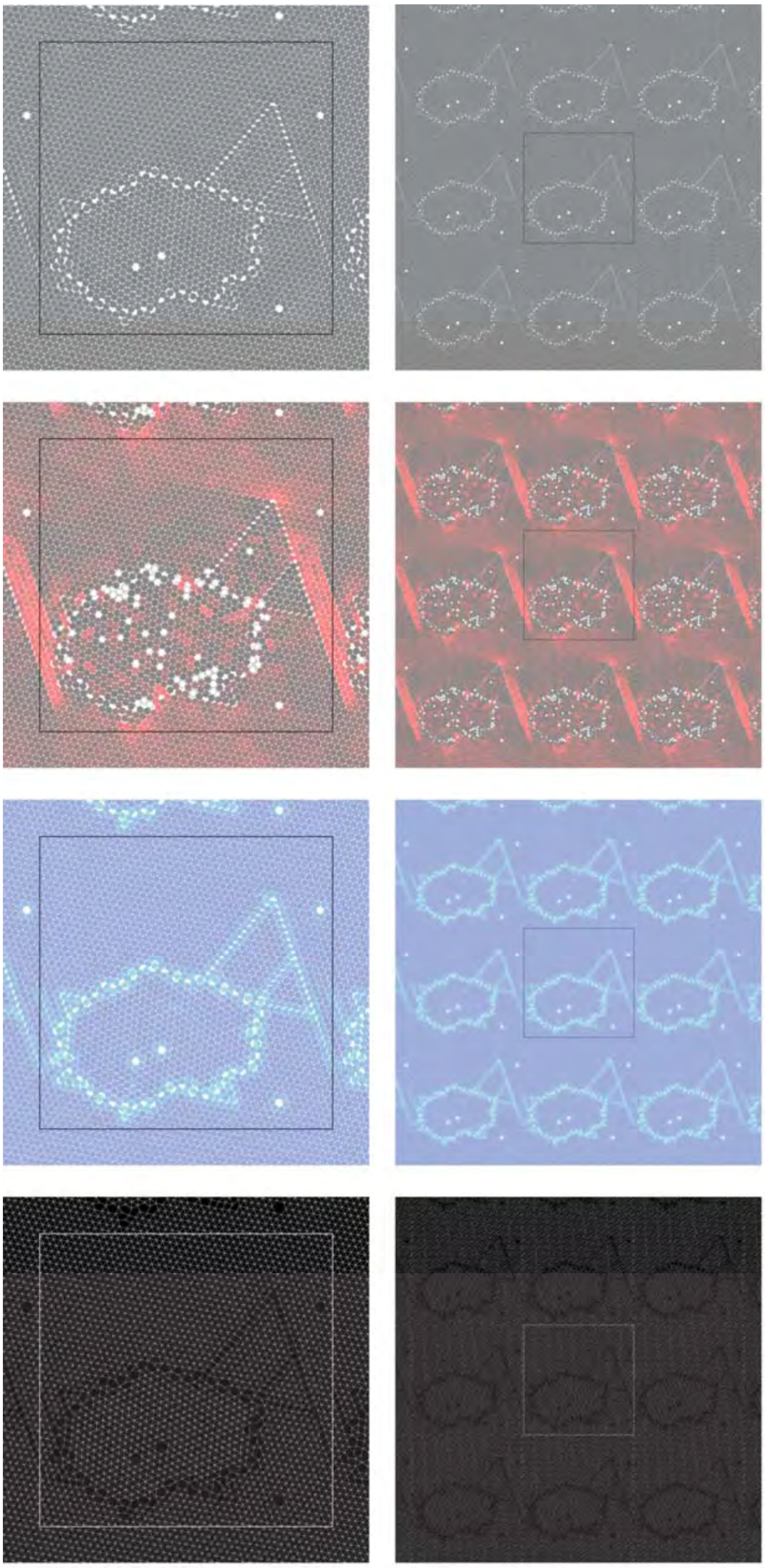


Fig. 20: "fish" wallpaper

Displays show the disks, uniformly coloured in grey, coloured accordingly to forces, to their number of neighbours, and lastly the links between neighbours.

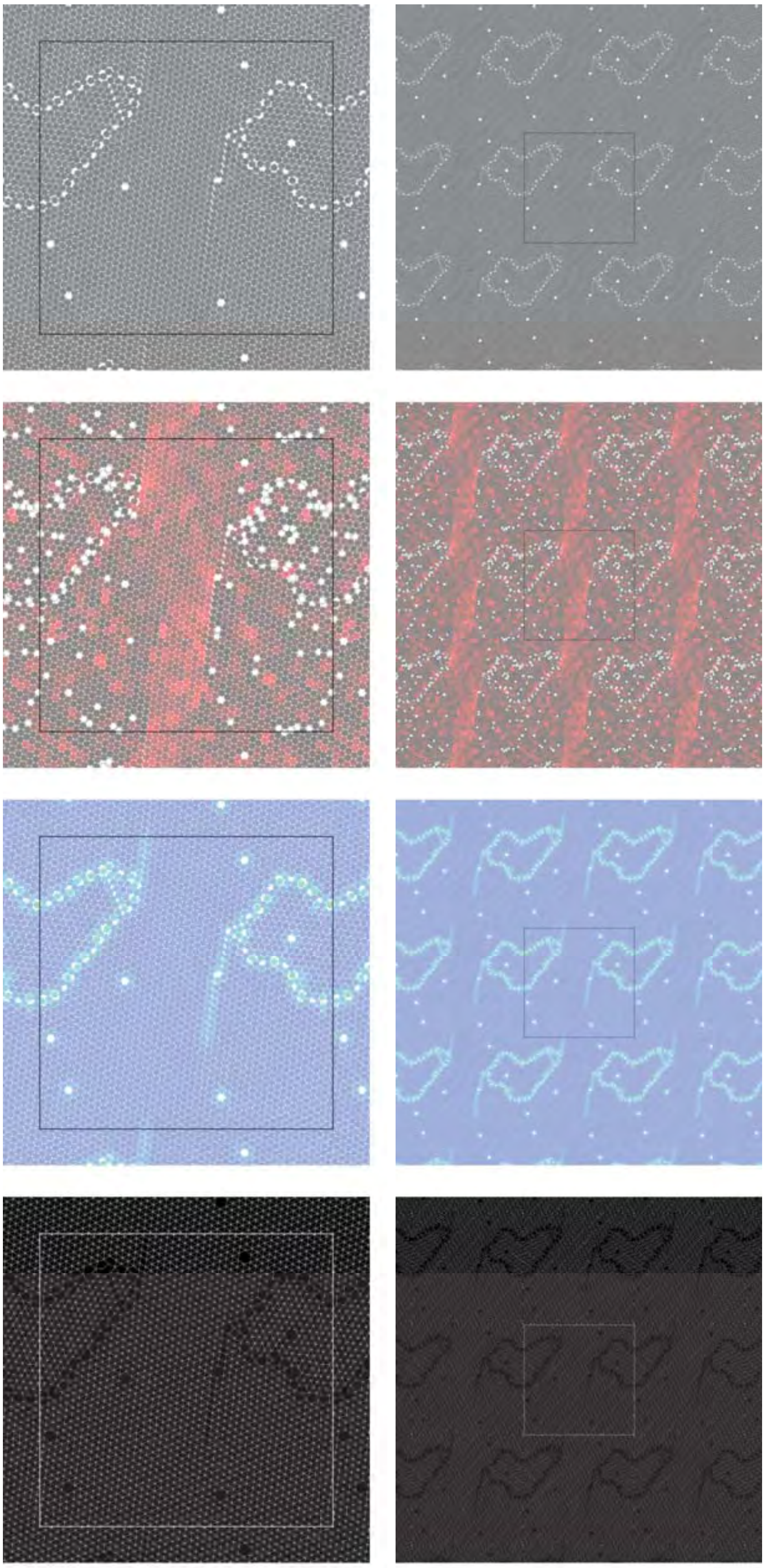


Fig. 21: "chick" wallpaper

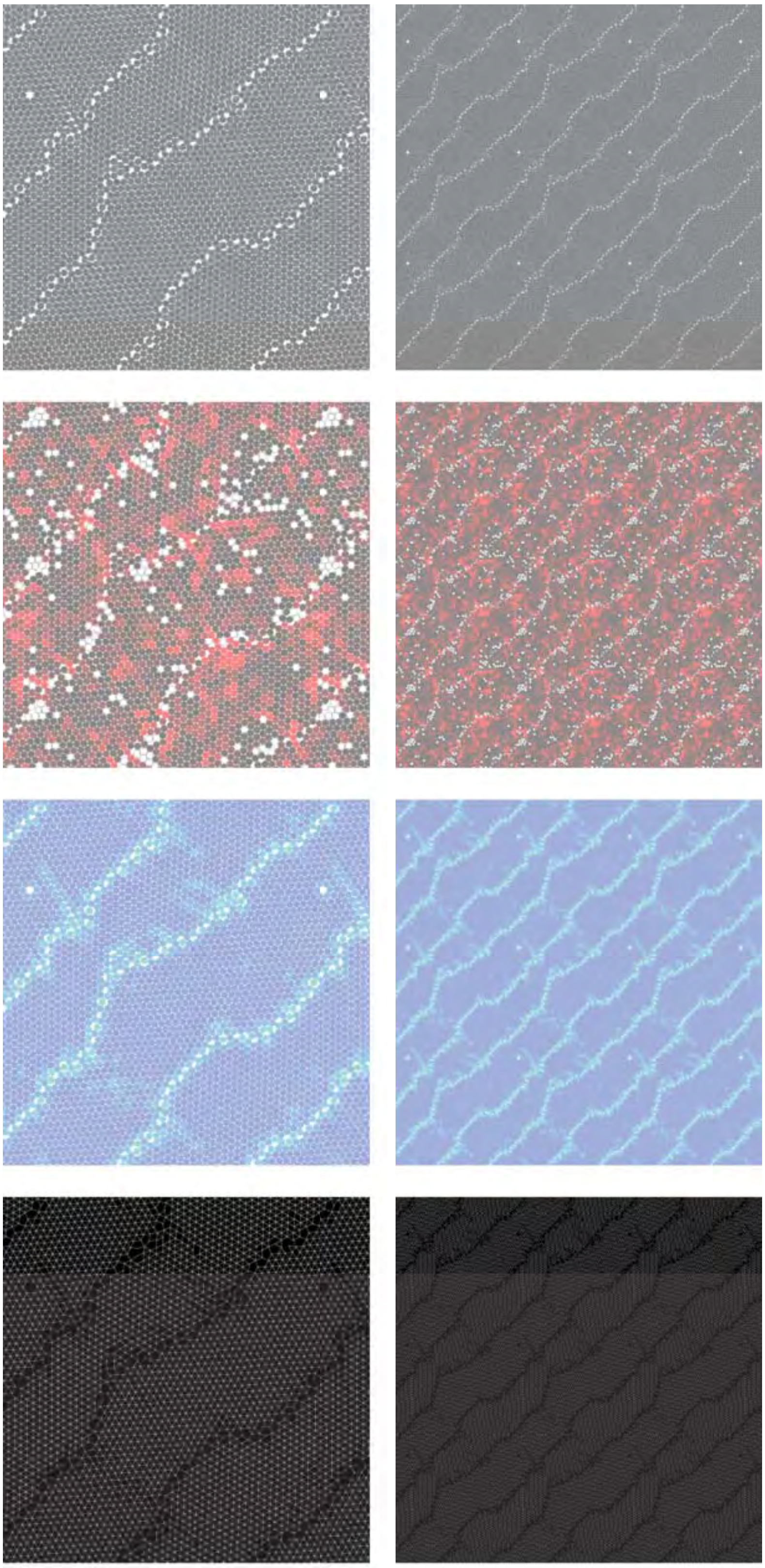


Fig. 22: "stripes" paperwall

Conclusion

Randomness is often a “trick” used to not to have to choose, to reduce the subjectivity, or to display one result among a huge number of possible results [6]. What we learned with the two models we explored is that in some cases, randomness, or disorder, is not only a convenience, but a *condition* of the evolution of the process. Ordered configurations do not evolve, they are stable, and what is even more interesting is that a very slight amount of disorder input in those ordered configurations is enough to perturb this stability.

Apart from their initial distibutions, both models are deterministic. In a way they are ways to display, to enhance, some random distribution of points in the plane, the discrete plane in the first case, the continuous one in the second. We tried to measure this disorder, by characterising not only the distribution of elements (which have the same probability to be in any location of the considered space) but their number of neighbours.

We do not pretend to contribute to a scientific study of randomness, disorder or entropy. At first glance, those models are counterintuitive in relation to a very loose knowledge of what entropy is, which we can express as: “entropy (or disorder) must allways increase”. But this is probably a too ingenuous view of those very difficult topics, and we forgot that it is only true for an isolated system. What we must acknowledge at least is that natural phenomena are well simulated by such processes. The Ising cellular automaton was designed to simulate the behaviour of spins in a ferromagnetic material, but its results suggest that it may simulate what happens in piebaldism, and maybe other phenomena as well. The disk packing model does not only simulate the actual behaviour of balls that you constrain on a shrinking surface, but may simulate what happens in crystallisation, where the random (Brownian) motion of molecules is replaced by a stable ordered configuration of them. What we see in actual crystal, is that contiguous ordered regions of different orientations emerge, as in the disk packing model.

According to wikipedia, “emergence is conceived as a process whereby larger entities, patterns, and regularities arise through interactions among smaller or simpler entities that themselves do not exhibit such properties.” [7] The two models we studied display such a behaviour, and we were able to precisely define and measure this arising of regularities. Beyond this objective measures, those patterns, and, in the case of the second model, the way they arise, are appealing to us (to some of us al least).

This allows to explain how those models ar able to contribute to the issues of generative art.

References

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